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# Optimal investment, consumption and timing of annuity purchase under a preference change

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ABSTRACT

In this paper, we study the optimal investment and consumption strategies for a retired individual who has the opportunity of choosing a discretionary stopping time to purchase an annuity. We assume that the individual receives a fixed annuity income and changes his/her preference after paying a fixed cost for annuitization. By using the martingale method and the variational inequality method, we tackle this problem and obtain the optimal strategies and the value function explicitly for the case of constant force of mortality and constant relative risk aversion (CRRA) utility function.

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## 1. Introduction

Since the seminal paper of Yaari [27], there have been a number of papers about the optimal annuitization and portfolio selection in the literature, see, for example, [9,22,24–26] and the references therein. Yaari [27] first demonstrated that under some specific assumptions rational individuals with no bequest motives should annuitize all their wealth at retirement. However, the volume of voluntary purchases by retirees is much smaller than predicted by theoretical models, which is the so-called "annuity puzzle".

Bequest motives play a central role in limiting the demand for annuities. Davidoff et al. [6] showed that if annuities are priced fairly, then people annuitize all of their wealth except what they wish to bequeath. Lockwood [21] argued that bequest motives are strong enough to reduce or eliminate purchases of available annuities. There are other explanations for the annuity puzzle. Inkmann et al. [11] found that the annuity market participation increases with financial wealth, life expectancy and education, while decreases with other pension income and a possible bequest motive for surviving spouses. Benartzi et al. [2] exploited that behavioral and institutional factors are important in explaining why there seems to be so little demand to annuitize wealth at retirement. Wang and Young [25,26] proposed that if life annuities were commutable, namely the individuals can surrender their early purchased life annuities, then retirees would purchase more annuities.

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Milevsky and Young [22] claimed that, as a result of adverse selection, the annuity purchase is an irreversible investment that creates an incentive to delay. Therefore, we consider a model which allows the retirees to prearrange their annuitization using part of their wealth in the future, the amount is fixed as a constant $F$. In view of significant medical spending, emergent events or bequest motives, full annuitization may not be optimal for the individual. Hence, we suppose that there is a nonnegative constant $d$ as the lowest wealth level around the annuitization time $\tau$, that is, $X_\tau \geqslant F + d$, where $X.$ is the wealth process of the individual.

In this paper, we investigate the optimal investment, consumption strategies and the optimal annuitization time for a retired individual subject to a constant force of mortality. The individual can determine a discretionary stopping time as the annuitization time, at that point, after paying a fixed cost for annuitization, the individual will receive a long-life annuity income. Moreover, we do not allow the individual to borrow his or her future annuity income because this income is contingent on his or her being alive. In contrast to the restrictive all-or-nothing arrangement, explored by [9,22,24], we assume that the individual continues investing and consuming and changes his or her preference after annuitization. Finally, four types of solutions are obtained depending on the free parameters of the problem. We find the optimal annuitization region is a band form. In some special cases, the two annuitization thresholds coincide or degenerate to $+\infty$.

The problem of utility maximization with discretionary stopping was first studied by Karatzas and Wang [16] via the martingale method. They introduced a family of stopping time problems to reduce the original problem into an easy form. Farhi and Panageas [8] applied these techniques to explore an optimal consumption and portfolio choice problem with flexible retirement option. See also [1,3–5,7,17,19,20] for other extensions. In this paper, we use both the martingale method and the variational inequality method to analyze an optimal consumption-portfolio selection problem with discretionary stopping. The wealth process in our model is assumed discontinuous. Another mixed optimal stopping/control problem has been studied by Jeanblanc et al. [12], in which the dynamic programming approach was employed and risk-preference change was not allowed.

The rest of our paper is organized as follows. In Section 2, we present our model and formulate the objective. An auxiliary value function $U(x)$ is introduced to transform the problem into an easily solved form. In Section 3, by using both the martingale method and the variational inequality method, we derive the closed-form solution for the auxiliary problem. In Section 4, the original value function and optimal strategies are provided by employing the same methods. Finally, we give numerical examples to illustrate our results in Section 5. Some of the detailed proofs are deferred to Appendices A and B.

## 2. Model formulation

### 2.1. The financial and pension annuity markets

We consider an optimization problem for an individual from the retirement time till the stochastic death time $\tau_d > 0$ in the financial and pension annuity markets. In the financial market, there are one risk-free asset and one risky asset, whose prices evolve according to the following equations:

$$dR_t = rR_t\,dt \quad \text{and} \quad dS_t = \mu S_t\,dt + \sigma S_t\,dW_t,$$

where $\mu$, $\sigma$ and $r$ are positive constants. $W_t$ is a standard Brownian motion on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and $\{\mathcal{F}_t\}$ is the $\mathbb{P}$-augmentation of the natural filtration generated by $W_t$. Suppose the death time $\tau_d$ is an exponential random variable with parameter $h$ defined on the probability space and is independent of $W_t$.

Let $c_t$ be the consumption rate process which is nonnegative, progressively measurable with respect to $\mathcal{F}_t$ satisfying $\int_0^t c_s\,ds < \infty$ a.s. for all $t \geqslant 0$, and $\pi_t$ be the amount invested in the risky asset at time $t$, which

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