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On the global regularity of two-dimensional generalized magnetohydrodynamics system

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ABSTRACT

We study the two-dimensional generalized magnetohydrodynamics system with dissipation and diffusion in terms of fractional Laplacians. In particular, in the case where the diffusion term has the power $\beta = 1$, in contrast to the previous result of $\alpha \geqslant \frac{1}{2}$, we show that $\alpha > \frac{1}{3}$ suffices in order for the solution pair of velocity and magnetic fields to remain smooth for all time.

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1. Introduction and statement of results

We study the generalized magnetohydrodynamics (MHD) system defined as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} + (u \cdot \nabla)u - (b \cdot \nabla)b + \nabla \pi + \nu \Lambda^{2\alpha} u = 0, \\ \dfrac{\partial b}{\partial t} + (u \cdot \nabla)b - (b \cdot \nabla)u + \eta \Lambda^{2\beta} b = 0, \\ \nabla \cdot u = \nabla \cdot b = 0, \quad u(x,0) = u_0(x),\ b(x,0) = b_0(x), \end{cases} \tag{1}$$

where $u : \mathbb{R}^N \times \mathbb{R}^+ \mapsto \mathbb{R}^N$ is the velocity vector field, $b : \mathbb{R}^N \times \mathbb{R}^+ \mapsto \mathbb{R}^N$ the magnetic vector field, $\pi : \mathbb{R}^N \times \mathbb{R}^+ \mapsto \mathbb{R}$ the pressure scalar field and $\nu, \eta \geqslant 0$ are the kinematic viscosity and diffusivity constants respectively. We also denote by Λ a fractional Laplacian operator defined via Fourier transform as $\widehat{\Lambda^{2\gamma} f}(\xi) = |\xi|^{2\gamma} \hat{f}(\xi)$ for any $\gamma \in \mathbb{R}$.

In the case $N = 2, 3$, $\nu, \eta > 0$, $\alpha = \beta = 1$, it is well-known that (1) possesses at least one global L^2 weak solution; in the case $N = 2$, it is also unique (cf. [20]). Moreover, in any dimension $N \geqslant 2$, in the case $\nu, \eta > 0$, the lower bounds on the powers of the fractional Laplacians at $\alpha \geqslant \frac{1}{2} + \frac{N}{4}$, $\beta \geqslant \frac{1}{2} + \frac{N}{4}$ imply the existence of the unique global strong solution pair (cf. [25]).

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Some numerical studies have shown that the velocity vector field may play relatively important role in regularizing effect (e.g. [8,19]). Starting from the works of [9] and [34], we have seen various regularity criteria of the MHD system in terms of only the velocity vector field (e.g. [1,4,6,7,10,26,29,35]). Moreover, motivated by the work of [21], the author in [27] showed that in the case $N \geqslant 2$, $\nu, \eta > 0$, $\alpha \geqslant \frac{1}{2} + \frac{N}{4}$, $\beta > 0$ such that $\alpha + \beta \geqslant 1 + \frac{N}{2}$, the system (1) even in logarithmically super-critical case still admits a unique global strong solution pair. The endpoint case $\nu > 0$, $\eta = 0$, $\alpha = 1 + \frac{N}{2}$ was also completed recently in [24] and [31] (cf. [28] for further generalization).

On the other hand, in the case $N = 2$, it is well-known that the Euler equations, the Navier–Stokes system with no dissipation, still admit a unique global strong solution. This is due to the conservation of vorticity which follows upon taking a curl on the system. In the case of the MHD system, upon taking a curl and then L^2-estimates of the resulting system, every non-linear term has b involved. Exploiting this observation and divergence-free conditions, the authors in [2] showed that in the case $N = 2$, full Laplacians in both dissipation and magnetic diffusion are not necessary for the solution pair to remain smooth; rather, only a mix of partial dissipation and diffusion in the order of two derivatives suffices.

Very recently, the authors in [23] have shown that in the case $N = 2$, the solution pair remains smooth in any of the following three cases:

(1) $\alpha \geqslant \frac{1}{2}$, $\beta \geqslant 1$,
(2) $\alpha \geqslant 2$, $\beta = 0$,
(3) $\frac{1}{2} > \alpha \geqslant 0$, $2\alpha + \beta > 2$.

In particular, their result implies that in the range of $\alpha \in [0, \frac{1}{2})$, β must satisfy

$$\beta > 2 - 2\alpha. \tag{2}$$

These results implied that if $\alpha = 0$, then $\beta > 2$ was necessary to obtain global regularity result. This was improved in [32] (cf. also [11]) to show that either of the following conditions suffices:

(1) $\alpha = 0$, $\beta > \frac{3}{2}$,
(2) $\frac{1}{2} > \alpha > 0$, $\frac{3}{2} \geqslant \beta > \frac{5}{4}$, $\alpha + 2\beta > 3$.

In particular, this implies that in the range of $\alpha \in (0, \frac{1}{2})$, β must satisfy

$$\beta > \frac{3 - \alpha}{2} \tag{3}$$

(cf. also [33]). In this paper we make further improvement in this direction. Let us present our results.

Theorem 1.1. *Let $N = 2$, $\nu, \eta > 0$, $\alpha > \frac{1}{3}$, $\beta = 1$. Then for all initial data pair $(u_0, b_0) \in H^s(\mathbb{R}^2) \times H^s(\mathbb{R}^2)$, $s \geqslant 3$, there exists a unique global strong solution pair (u, b) to (1) such that*

$$u \in C\big([0,\infty); H^s(\mathbb{R}^2)\big) \cap L^2\big([0,\infty); H^{s+\alpha}(\mathbb{R}^2)\big),$$
$$b \in C\big([0,\infty); H^s(\mathbb{R}^2)\big) \cap L^2\big([0,\infty); H^{s+1}(\mathbb{R}^2)\big).$$

Theorem 1.2. *Let $N = 2$, $\nu, \eta > 0$, $\alpha \in (0, \frac{1}{3}]$, $\beta \in (1, \frac{3}{2}]$ such that*

$$3 < 2\beta + \frac{2\alpha}{1 - \alpha}. \tag{4}$$

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