J. Math. Anal. Appl. 416 (2014) 212–228

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

A new approach to the stability of an abstract system in the presence of infinite history

A. Guesmia [a,*], S.A. Messaoudi [b]

[a] Institut Elie Cartan de Lorraine, UMR 7502, Université de Lorraine, Bat. A, Ile de Saulcy, 57045 Metz Cedex 01, France
[b] Department of Mathematics and Statistics, KFUPM, Dhahran 31261, Saudi Arabia

ARTICLE INFO

Article history:
Received 4 February 2013
Available online 20 February 2014
Submitted by K. Nishihara

Keywords:
Infinite history
General decay
Viscoelasticity

ABSTRACT

In this paper, we consider the following problem

$$\begin{cases} u_{tt}(t) + Au(t) - \int_0^{+\infty} g(s)Au(t-s)\,ds = 0, & \forall t > 0 \\ u(-t) = u_0(t), & \forall t \geqslant 0 \\ u_t(0) = u_1, \end{cases}$$

where A is a self-adjoint positive definite operator and g is a positive nonincreasing function. We adopt the method introduced in [19], for finite history, with some modifications imposed by the nature of our problem, to establish a general decay result which depends only on the behavior of the relaxation function. Our result extends the decay result obtained for problems with finite history to those with infinite history. In addition, it improves, in some cases, some decay results obtained earlier in [15].

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

Let H be a real Hilbert space with inner product and related norm denoted by $\langle \cdot, \cdot \rangle$ and $\|\cdot\|$, respectively. Let $A : D(A) \to H$ be a self-adjoint linear positive definite operator with domain $D(A) \subset H$ such that the embedding is dense and compact. We consider the following class of second-order linear integrodifferential equations:

* Corresponding author. Present address: Department of Mathematics and Statistics, KFUPM, Dhahran 31261, Saudi Arabia.
E-mail addresses: guesmia@univ-metz.fr, guesmia@kfupm.edu.sa (A. Guesmia), messaoud@kfupm.edu.sa (S.A. Messaoudi).

http://dx.doi.org/10.1016/j.jmaa.2014.02.030
0022-247X/© 2014 Elsevier Inc. All rights reserved.

$$u_{tt}(t) + Au(t) - \int_0^{+\infty} g(s)Au(t-s)\,ds = 0, \quad \forall t > 0 \tag{1.1}$$

with initial conditions

$$\begin{cases} u(-t) = u_0(t), \quad \forall t \in \mathbb{R}^+ = [0, +\infty[\\ u_t(0) = u_1 \end{cases} \tag{1.2}$$

where $u_{tt} = \frac{\partial^2 u}{\partial t^2}$, $u_t = \frac{\partial u}{\partial t}$, u_0 and u_1 are given history and initial data, and $g : \mathbb{R}^+ \to \mathbb{R}^+$ is a given function.

Since the pioneer work of Dafermos [10], problems related to (1.1)–(1.2) have attracted the attention of many researchers and a large number of papers have appeared. We start by the work of Chepyzhov and Pata [9], where an abstract problem of the form

$$\begin{cases} u_{tt}(t) + Au(t) - \int_0^{+\infty} g(s)\big(A(t) - Au(t-s)\big)\,ds = 0, & \forall t > 0 \\ u(-t) = u_0(t), & \forall t \geqslant 0 \\ u_t(0) = u_1 \end{cases}$$

was considered in a Hilbert space H. Here A is a strictly positive self-adjoint operator with a domain $D(A) \subset H$ and $0 < \int_0^{+\infty} g(s)\,ds < +\infty$. They proved the well-posedness and showed that the exponential stability holds only for kernels of exponential decay. Also, in a survey paper, Pata [28] discussed the decay properties of the semigroup associated with Eq. (1.1) and established several stability results. In [27], Pata studied the asymptotic behavior of an abstract integrodifferential equation of the form

$$u_{tt}(t) + \alpha Au(t) + \beta u_t - \int_0^{t} g(s)Au(t-s)\,ds = 0, \quad \forall t > 0$$

for $\alpha > 0$, $\beta \geqslant 0$ and g a positive summable kernel, and analyzed the exponential stability of the semigroup associated with the positive operator under some sufficient conditions on the kernel which were not considered before in the literature. He also introduced some new concepts such as the flatness of a kernel. We refer the reader to Fabrizio et al. [13] and Grasselli et al. [14] for more results of this nature.

In all the above mentioned works, the kernels considered were of either exponential or polynomial decay. Recently, Guesmia [15] considered the following problem:

$$\begin{cases} u_{tt}(t) + Au(t) - \int_0^{+\infty} g(s)Bu(t-s)\,ds = 0, & \forall t > 0 \\ u(-t) = u_0(t), & \forall t \geqslant 0 \\ u_t(0) = u_1 \end{cases}$$

for A and B two self-adjoint positive definite operators with $D(A) \subset D(B)$ and a kernel of more general decay rate satisfying

(A0) There exist $a_0, a_1 > 0$ such that

$$a_1\|v\|^2 \leqslant \big\|B^{\frac{1}{2}}v\big\|^2 \leqslant a_0\big\|A^{\frac{1}{2}}v\big\|^2, \quad \forall v \in D(A)$$

Download English Version:

https://daneshyari.com/en/article/4616153

Download Persian Version:

https://daneshyari.com/article/4616153

Daneshyari.com