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Analytical pricing of defaultable discrete coupon bonds in unified two-factor model of structural and reduced form models

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ABSTRACT

Pricing formulae for defaultable corporate bonds with discrete coupons (under consideration of the government taxes) in the united two-factor model of structural and reduced form models are provided. The aim of this paper is to generalize the two-factor structural model for defaultable corporate discrete coupon bonds (considered in [1]) into the unified model of structural and reduced form models. In our model the bond holders receive the stochastic coupon (which is the discounted value of a predetermined value at the maturity) at predetermined coupon dates and the face value (debt) and the coupon at the maturity as well as the effect of government taxes which are paid on the proceeds of an investment in bonds is considered. The expected default event occurs when the equity value is not sufficient to pay coupon or debt at the coupon dates or maturity and the unexpected default event can occur at the first jump time of a Poisson process with the given default intensity provided by a step function of time variable. We provide the model and pricing formula for equity value and using it calculate expected default barrier. Then we provide pricing model and formula for defaultable corporate bonds with discrete coupons and consider its duration.

1. Introduction

The study on defaultable corporate bonds and credit risk is now one of the most promising areas of cutting edge in financial mathematics [1].

As is well known, there are two main approaches to pricing defaultable corporate bonds; one is the *structural approach* and the other one is the *reduced form approach*. In the structural method, we think that the default event occurs when the firm value is not sufficient to repay debt, that is, the firm value reaches a certain lower threshold (*default barrier*) from the above. Such a default can be expected and thus we call it *expected default*. In the reduced-form approach, the default is treated as an unpredictable event governed by a default intensity process. In this case, the default event can occur without any correlation with the firm

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value and such a default is called *unexpected default*. In the reduced-form approach, if the default probability in time interval $[t, t + \Delta t]$ is $\lambda \Delta t$, then λ is called *default intensity* or *hazard rate*. The third approach is to unify the structural and reduced form approaches. As for the history of the above three approaches and their advantages and shortcomings, readers can refer to [14] and the introductions of [3,4,13]. Combining the elements of the structural approach and reduced-form approach is one of the recent trends [3].

On the other hand, many models related to coupon approximate actual coupon bearing debts with continuous coupon stream or even zero coupon contracts, but such approach has restriction [6].

There has been relatively little work on the most realistic payout structure providing fixed discrete coupons [1]. Geske [5] is the first study for this problem, where discrete interest payouts prior to maturity were modeled as determinants of default risk [5]. Recently, Agliardi [1] generalized Geske's formula for defaultable coupon bonds, incorporated a stochastic risk free term structure and the effects of bankruptcy cost and government taxes on bond interest and studied the duration of defaultable bonds. The introduction and the conclusions of [1] include useful information about corporate discrete coupon bonds. Agliardi's approach in [1] to corporate coupon bonds is a kind of structural approach as shown in its title. In [10], authors studied some general properties of solutions to inhomogeneous Black–Scholes equations with discontinuous maturity payoffs and applied them to a pricing problem of defaultable discrete coupon bond with constant default intensity in a unified model of structural and reduced form models. Unlike [1], the authors of [10] calculated the expected barrier from the bond price.

The purpose of this article is to generalize the results of [1] into a unified model of structural and reduced form models. In our model the bond holders receive the *stochastic* coupon (*discounted value of that at the maturity*) at predetermined coupon dates and the face value (debt) and the coupon at the maturity, which is different from [1] and the aim of such a change of coupon structure is to get analytical pricing formulae. The effect of government taxes which are paid on the proceeds of an investment in bonds is considered. The expected default event occurs when the *equity value* is not sufficient to pay coupon or debt at the coupon dates or maturity and unexpected default event can occur at the first jump time of a Poisson process with the given default intensity provided by a step function of time variable. Here we consider the corporate bond as a derivative of the firm value and default free short rate (in order to avoid possible confusion) and the coupon prior to the maturity as a discounted value of the maturity-value (in order to obtain analytical pricing formulae). In [1] they considered the corporate bond as a derivative of the firm value and default free zero coupon bonds with several maturities (although they considered the corporate bond as a derivative of the firm value and default free short rate in study of duration). We consider the model and pricing formula for equity value and using it calculate expected default barrier. Then we provide pricing model and formula for defaultable discrete coupon corporate bonds and consider its duration.

In our case, the pricing model between every adjacent two coupon dates is derived to an inhomogeneous Black–Scholes equation with discontinuous terminal value condition and can be changed into the problem which can be solved by the method of higher order binaries which is used to the pricing problem of corporate zero coupon bonds in [12] or [9].

The remainder of the article is organized as follows. In Section 2 we consider the model and pricing formula for equity value and using it calculate expected default barrier. Then we provide pricing model and formula for defaultable discrete coupon corporate bonds without consideration of taxes. In Section 3 we study duration of our bond. In Section 4 we consider the effect of taxes. In Appendix A we provide the pricing formulae of higher order binaries with time dependent coefficients and their some properties which are used in the previous sections. In Appendix B we give the sketch of the proof of pricing formulae for equity value.