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On the Fock space of metaanalytic functions

Yingxiong Fu [a,b], Luoqing Li [a,*], Uwe Kaehler [b], Paula Cerejeiras [b]

[a] Faculty of Mathematics and Computer Science, Hubei University, Hubei, China
[b] Department of Mathematics, University of Aveiro, Portugal

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ABSTRACT

We develop the theory on the Fock space of metaanalytic functions, a generalization of some recent results on the Fock space of polyanalytic functions. We show that the metaanalytic Bargmann transform is a unitary mapping between vector-valued Hilbert spaces and metaanalytic Fock spaces. A reproducing kernel of the metaanalytic Fock space is derived in an explicit form. Furthermore, we establish a complete characterization of all lattice sampling and interpolating sequence for the Fock space of metaanalytic functions.

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1. Introduction

A function $f(z)$, $z = x + i\omega$, that has continuous partial derivative with respect to x and ω up to order $n \geqslant 1$ on the whole complex plane $\mathbb{C}$ is called a polyanalytic function on $\mathbb{C}$ if it satisfies the generalized Cauchy–Riemann equation

$$\frac{\partial^n f}{\partial \bar{z}^n} = 0, \quad \forall z \in \mathbb{C}, \tag{1.1}$$

where the Cauchy–Riemann operator is defined by

$$\frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial \omega}\right). \tag{1.2}$$

Polyanalytic functions inherit some of the properties of analytic functions, often in a nontrivial form. However, many of the properties break down once we leave the analytic setting. A clear difference lies in the structure of the zeros. A theory on polyanalytic functions had been investigated thoroughly, notably by the Russian school led by Balk [5], and provided extensions of the classical operators from complex analysis [6].

* Corresponding author.
E-mail addresses: fyx@hubu.edu.cn (Y. Fu), lilq@hubu.edu.cn (L. Li), ukaehler@ua.pt (U. Kaehler), pceres@ua.pt (P. Cerejeiras).

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It is well known that there exists a close connection between the classical Bargman–Fock space of analytic functions and the time-frequency analysis. Specifically, up to a certain weight, the Gabor transform (also called short-time Fourier transform or windowed Fourier transform) with a Gaussian window is tightly related to the Bargmann transform and the corresponding Fock space of analytic functions [10]. Moreover, it had been shown that this is the only choice leading to spaces of analytic functions [4].

Recently, a series of works have been devoted to the theory of the vector-valued Gabor frames (sometimes called Gabor superframe) with Hermite functions [1,9,11,12]. It is worth pointing out that a new connection between polyanalytic functions and time-frequency analysis is established. Actually, the Gabor transform with the kth Hermite function is shown to be a polyanalytic function of order $k+1$, which is corresponding to the true polyanalytic Bargmann transform of order k, a unitary mapping between the Hilbert spaces and true polyanalytic Fock spaces [1–3]. The polyanalytic Fock space of order n can be orthogonally decomposed into a superposition of all true polyanalytic Fock spaces up to order $n-1$. Moreover, the concept of interpolating and sampling sequences corresponds to the case where stable numerical reconstruction of a function from its samples is possible [13]. A complete characterization of all lattice sampling and interpolating sequence was developed for the Fock space of polyanalytic functions, or equivalently, for all vector-valued Gabor frames with Hermite functions [1]. Furthermore, in [2], the authors studied the structure of Gabor and super-Gabor space and obtained an explicit formula of reproducing kernel for the Fock space of polyanalytic functions.

As mentioned briefly in Balk's monography [5], polyanalytic functions can be extended to more general cases, such as metaanalytic functions. Metaanalytic functions are closely connected with many applications in mathematics and physics, some works on them had captured the attention of many researchers [8,14–16].

In this paper, our original goal is to deal with the Fock space of metaanalytic functions, which is a generalization of the Fock space of polyanalytic functions, recently proposed by Abreu [1,2]. We establish the definition of the metaanalytic Bargmann transform and prove it to be a unitary mapping between vector-valued Hilbert spaces and metaanalytic Fock spaces. Moreover, we obtain an explicit formula for the reproducing kernel of the metaanalytic Fock space from which growth estimates can be derived. As for sampling and interpolating in the Fock space of metaanalytic functions, we show them to be equivalent to the cases in the Fock space of polyanalytic functions.

This paper is organized as follows: Section 2 is devoted to reviewing some definitions and basic properties of metaanalytic functions and the Bargmann transform. In Section 3, we provide the definitions of the true metaanalytic Bargmann transform and metaanalytic Bargmann transform, and show some properties of them. In Section 4, we proceed with the study of the reproducing kernel in the metaanalytic Fock space. Furthermore, in Section 5, we establish a complete characterization of all lattice sampling and interpolating sequence for the Fock space of metaanalytic functions.

2. Preliminaries

2.1. Metaanalytic functions

To generalize the definition of polyanalytic function given by (1.1), let

$$\mathcal{M}_n := \sum_{k=0}^{n} \binom{n}{k} (-\lambda)^{n-k} \frac{\partial^k}{\partial \bar{z}^k} = \left(\frac{\partial}{\partial \bar{z}} - \lambda \right)^n \tag{2.1}$$

be a polynomial of Cauchy–Riemann operator $\frac{\partial}{\partial \bar{z}}$ defined by (1.2), where λ is a complex constant.

Definition 2.1. A function $f(z)$ that has continuous partial derivatives with respect to x and ω up to order $n \geqslant 1$ is called a metaanalytic function of order n on $\mathbb{C}$ if it satisfies $\mathcal{M}_n f = 0$ on $\mathbb{C}$, denoted by $f \in \mathcal{M}_n(\mathbb{C})$, where the operator $\mathcal{M}_n$ is defined by (2.1).

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