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A class of integral operators on spaces of analytic functions

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A R T I C L E I N F O

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We determine the spectrum and essential spectrum as well as resolvent estimates for a class of integral operators

$$T_{\mu,\nu}f(z) = z^{\mu-1}(1-z)^{-\nu}\int_0^z f(\xi)\xi^{-\mu}(1-\xi)^{\nu-1}\,d\xi$$

acting on either the analytic Besov spaces or other Banach spaces of analytic functions on the unit disc, including the classical Hardy and weighted Bergman spaces as well as certain Dirichlet spaces and generalized Bloch spaces. Our results unify and extend recent work by Aleman and Persson, [2], the current authors, [4], and Albrecht and Miller, [1].

1. Introduction

Investigation of the classical Cesàro operator on Hardy and Bergman spaces culminated in work by Dahlner, [5], and Persson, [9], which provided a complete spectral picture in the setting of H^p, $p \geqslant 1$ and $L_a^{p,\alpha}$ for $p \geqslant 1$ for weights corresponding to $\alpha \geqslant 0$ and, $p > 1$ when $-1 < \alpha < 0$. These results have been generalized and extended in recent papers [4] and [2].

Specifically, [4] addresses integral operators on weighted Bergman spaces of the form

$$C_\nu f(z) = \frac{1}{z}\int_0^z f(\xi)\xi^{\nu-1}(1-\xi)^{-1}\,d\xi.$$

These operators are related to semigroups introduced by Siskakis, [10], in his proof of boundedness of the Cesàro operator on the Hardy spaces H^p, $p > 1$. The focus of [2] is on generalized Cesàro operators

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$$C_g f(z) = \frac{1}{z} \int_0^z f(\xi) g'(\xi)\, d\xi$$

with essentially rational g' on very general Banach spaces of analytic functions detailed below. In particular, resolvents of these generalized Cesàro operators were given in terms of operators of the form

$$T_{\mu,\nu} f(z) = z^{\mu-1}(1-z)^{-\nu} \int_0^z f(\xi)\xi^{-\mu}(1-\xi)^{\nu-1}\, d\xi;$$

see [2, Lemma 4.1]. In [1], spectral properties of a related operator,

$$Q_\mu f(z) = (1-z)^{\mu-1} \int_0^z f(\xi)(1-\xi)^{-\mu}\, d\xi$$

were obtained in the setting of the analytic Besov spaces, B_p, $p < \infty$, and the little Bloch space. Since these spaces do not satisfy Aleman and Persson's assumptions (A-1) and (A-2) below, different methods were required in [1]. Moreover, because density of the polynomials played an essential role in [1], spectral properties of Q_μ on the little Bloch space could not be extended to the entire Bloch space.

The operators Q_μ and C_ν above are closely related to the operators $T_{\mu,\nu}$ employed by Aleman and Persson: $C_\nu = T_{1-\nu,0}$ and $Q_\mu = M_z T_{0,1-\mu}$, where M_z denotes multiplication by the independent variable. The purpose of this paper is to first complete the spectral analysis of the operators $T_{\mu,\nu}$ on the spaces under consideration in [2] and then in the setting of the analytic Besov spaces.

For an operator T on a complex Banach space X, let $\sigma(T,X)$ and $\sigma_p(T,X)$ denote the spectrum and point spectrum of T, respectively. The kernel and range of T are denoted respectively $\ker T$ and $\operatorname{ran} T$. Let $\mathcal{L}(X)$ denote the algebra of bounded linear operators on X and, if $T \in \mathcal{L}(X)$, denote the approximate point spectrum by $\sigma_{ap}(T,X)$. The surjectivity spectrum of T is $\sigma_{su}(T,X) = \{\lambda\colon\ \lambda - T \text{ is not surjective}\}$ and the essential spectrum is $\sigma_e(T,X) = \{\lambda\colon\ \lambda - T \text{ is not Fredholm}\}$. For each of these spectra $\sigma_\bullet(T,X)$, let $\rho_\bullet(T,X) := \mathbb{C} \setminus \sigma_\bullet(T,X)$ denote the corresponding resolvent set. When there is no chance of ambiguity, we write simply $\sigma_\bullet(T)$ rather than $\sigma_\bullet(T,X)$, etc. We denote by $B(a,r)$ the open ball centered at a with radius r.

Let $\mathbb{C}[z]$ denote the vector space of analytic polynomials. If $E \subset \mathbb{C}$ let $\mathcal{H}(E)$ be the space of $\mathbb{C}$-valued functions f analytic on an open neighborhood U_f of E; in particular, if $\mathbb{D}$ denotes the unit disc, $\mathbb{D} = \{z\colon\ |z| < 1\}$, then $\mathcal{H}(\mathbb{D})$ is the Fréchet space of functions analytic on $\mathbb{D}$, with topology generated by the seminorms $\|f\|_K = \sup_{z\in K} |f(z)|$, $K \subset \mathbb{D}$ compact. For $a \in \mathbb{D}$, define

$$\phi_a(z) = \frac{a-z}{1-\bar{a}z},$$

so that the group of automorphisms of $\mathbb{D}$ is $\{c\phi_a\colon\ a \in \mathbb{D}, |c| = 1\}$.

For $\gamma > 0$, the generalized Bloch space B_∞^γ consists of functions $f \in \mathcal{H}(\mathbb{D})$ such that the seminorm

$$\|f\|_{B_\infty^\gamma,1} := \sup_{z\in\mathbb{D}} \left(1-|z|^2\right)^\gamma \left|f'(z)\right| < \infty.$$

By [13, Proposition 1], each B_∞^γ is a Banach space with respect to the norm $\|f\|_{B_\infty^\gamma} := |f(0)| + \|f\|_{B_\infty^\gamma,1}$; moreover, by [13, Proposition 7], $f \in \mathcal{H}(\mathbb{D})$ is in $B_\infty^{1+\gamma}$ if and only if

$$\sup_{z\in\mathbb{D}} \left(1-|z|^2\right)^\gamma \left|f(z)\right| < \infty,$$