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On the existence and the uniqueness of the solution of a fluid–structure interaction scattering problem

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ABSTRACT

The existence and uniqueness of the solution of a fluid–structure interaction problem is investigated. The proposed analysis distinguishes itself from previous studies by employing a weighted Sobolev space framework, the DtN operator properties, and the Fredholm theory. The proposed approach allows to extend the range of validity of the standard existence and uniqueness results to the case where the elastic scatterer is assumed to be only Lipschitz continuous, which is of more practical interest.

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1. Introduction

The mathematical analysis and the numerical computation of scattered fields by penetrable objects are very important to many real-world applications such as radar and sonar detection, geophysical exploration, structural design, medical imaging, and atmospheric studies. The goal of the proposed study is to investigate the well-posed nature of a class of elasto-acoustic scattering problems that describes the propagation of a scattered field from an elastic bounded object immersed in an infinite domain, representing a fluid medium. This class of problems consists in the coupling of Helmholtz equation with Navier equation.

Helmholtz problems, per se, have been analyzed extensively from both mathematical and numerical viewpoints, and results pertaining to existence and uniqueness can be found in [9,37,40,49], among others. Likewise, elastic scattering problems have been also investigated mathematically and numerically, and results pertaining to their well-posedness can be found in [34,35,41,44]. However, there have been relatively very few mathematical works on problems involving the coupling of Helmholtz and Navier equations. Indeed, to the best of our knowledge, the well-posed nature of the coupling system has been studied first in [38] and then a few years later in [30]. In reference [38], the authors reformulated the considered boundary value problem as an integro-differential system whose unknowns are defined on the fluid–structure interface Γ. Such a transformation was accomplished using an integral representation of both the fluid pressure and structural displacement fields. In doing this, the authors established existence and uniqueness results assuming the boundary Γ of the scatterer to be $\mathcal{C}^2$, which is a very restrictive condition when considering practical situations. In reference [30], the

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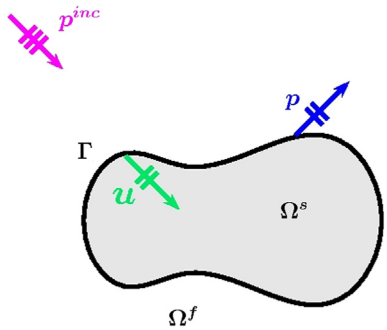

Fig. 1. Problem statement in the infinite domain.

authors adopted a different approach that relies on the integral representation of the fluid pressure only. Yet, the approach requires also a $\mathcal{C}^2$ regularity on the boundary Γ to establish similar existence and uniqueness results. Note that this formulation has been numerically implemented in [16] for solving the corresponding inverse elasto-acoustic scattering problem. We must point out that the formulation employed in [16] is slightly different than the adopted in [30]. The authors in [16] consider an artificial exterior boundary surrounding the elastic scatterer, on which an exact boundary condition is imposed via the integral formulation of the fluid pressure.

We propose here to extend the results obtained in [38] and [16] to the case where the wet surface Γ is assumed to be only Lipschitz continuous, which is of more practical interest. The proposed proof employs a weighted Sobolev space framework [20,26], the Dirichlet-to-Neumann (DtN) operator [21,24,47], the Gårding inequality [5,43,45], and the Fredholm alternative [5,43,45]. More specifically, the proposed proof can be viewed as a four-step approach:

- In step 1, we specify the mathematical framework for the considered boundary value problem (BVP). We construct a weighted Sobolev-like space that naturally incorporates the asymptotic decay of the fluid pressure variable p as well as its outgoing propagation nature.
- Step 2 consists in reformulating the BVP in a bounded domain. Unlike the approach used in [30], we prescribe the exact DtN boundary condition at the exterior spherical-shaped boundary. Note that adopting the weighted Sobolev space framework and the DtN operator allows to rigorously establish the equivalence between both boundary value problems. To the best of our knowledge, the equivalence between the BVP and the formulation in the bounded domain is established rigorously for the first time.
- Step 3 focuses on the boundary value problem formulated in a finite domain. We derive a variational formulation for this problem, and then, using the sign property of the DtN operator, we prove that the Gårding's inequality holds.
- Step 4 consists in applying the Fredholm alternative which allows to prove, under minimal condition on the regularity of the fluid–structure interface Γ, (a) the existence of the solution of the BVP, (b) the uniqueness of the fluid pressure, and (c) the uniqueness of the structural displacement field modulo Jones frequencies [13,34]. These frequencies may exist only for a particular class of elastic objects, such as spheres [13,17,29].

The remainder of the paper is organized as follows. In Section 2, we first state the considered mathematical model in the infinite domain. Then, we introduce the weighted Sobolev space formulation and the formulation in a bounded domain. Finally, we prove the equivalence between the two formulations. Section 3 is devoted to the mathematical analysis of the boundary value problem formulated in a bounded domain. More specifically, we state the variational formulation corresponding to this problem. We then establish the equivalence between the strong and the weak formulations, and then examine the properties of the considered variational problem. In Section 4, we investigate the existence and the uniqueness of the solution of the resulting variational problem. Using the Fredholm alternative, we prove the existence of the solution. We then prove that the pressure field is unique, whereas the displacement field is unique modulo Jones frequencies. Appendix A contains standard analytical results pertaining to the solution of the exterior Helmholtz problems and the corresponding DtN mapping in the case of the spherical coordinates system. These results are included only for completeness.

2. The boundary value problem formulations

2.1. Formulation in the infinite domain

Let Ω^s be a bounded domain of $\mathbb{R}^3$ representing an elastic obstacle, and $\Omega^f = \mathbb{R}^3 \setminus \overline{\Omega^s}$ be the homogeneous inviscid (fluid) medium surrounding the elastic domain. Γ is the boundary of Ω^s and is assumed to be Lipschitz continuous.

We consider the scattering of a time-harmonic acoustic wave by the elastic obstacle Ω^s embedded in Ω^f as depicted in Fig. 1. The corresponding system of equations BVP (1) reads as the coupling of the Helmholtz and Navier equations. This problem can be formulated as follows: