J. Math. Anal. Appl. 412 (2014) 989–997

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Anticipated backward stochastic differential equations driven by the Teugels martingales

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ARTICLE INFO

Article history:
Received 7 November 2012
Available online 11 November 2013
Submitted by U. Stadtmueller

Keywords:
Anticipated backward stochastic differential equations
Lévy process
Teugels martingales
Comparison theorem

ABSTRACT

In this paper, a class of anticipated backward stochastic differential equations driven by Teugels martingales associated with Lévy process is investigated. We obtain the existence and uniqueness of solutions to these equations by means of the fixed-point theorem. We show that a comparison theorem for this type of ABSDEs also holds under some slight stronger conditions.

1. Introduction

Backward stochastic differential equations (BSDEs in short) were introduced by Bismut for the linear case [2] and by Pardoux and Peng in the general case [8]. Precisely, according to [8], given a data (ξ, f) consisting of a square integrable random variable ξ and a progressively measurable process f, a so-called generator, they proved the existence and uniqueness of a solution to the following BSDE:

$$Y(t) = \xi + \int_t^T f\big(s, Y(s), Z(s)\big)\,ds - \int_t^T Z(s)\,dW(s), \quad 0 \leqslant t \leqslant T,$$

where W is a Brownian motion. Backward stochastic differential equations have attracted great interest due to their connections with stochastic optimal control and stochastic games [4], mathematical finance [3]. Backward stochastic differential equations also appear as a powerful tool in partial differential equations where they provide probabilistic formulas for their solutions [9,10].

Recently, a new type of BSDE, called anticipated BSDE (ABSDE in short), which can be regarded as a new duality type of stochastic differential delay equations, was introduced by [11], see also [15]. The ABSDE is of the form

$$\begin{cases} -dY(t) = f\big(t, Y(t), Z(t), Y\big(t+\mu(t)\big), Z\big(t+\nu(t)\big)\big)\,dt - Z(t)\,dW(t), & t \in [0, T], \\ Y(t) = \xi(t), \qquad Z(t) = \eta(t), & t \in [T, T+K], \end{cases} \tag{1}$$

where $\mu(\cdot) : [0, T] \to \mathbb{R}^+ \setminus \{0\}$ and $\nu(\cdot) : [0, T] \to \mathbb{R}^+ \setminus \{0\}$ are continuous functions.

In [6], a martingale representation theorem associated to Lévy processes was proved. Then it is natural to extend BSDEs driven by Brownian motion to BSDEs driven by a Lévy process [7]. In their paper, based on this representation theorem, the

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0022-247X/$ – see front matter
http://dx.doi.org/10.1016/j.jmaa.2013.11.009

authors proved the existence and uniqueness of solutions of BSDEs driven by Teugels martingales associated with a Lévy process, under Lipschitz conditions on the generator f. These results were important from a pure mathematical point of view as well as from an application point of view in the world of finance. Specifically, they could be used for the purpose of option pricing in a Lévy market and related PDE which provided an analogue of the famous Black–Scholes formula.

Since then, the theory of BSDEs driven by a Lévy process has been widely studied. Ref. [13] proposed reflected BSDEs driven by a Lévy process and derived the existence and uniqueness of the solution. Ref. [12] gave a probabilistic formula for the viscosity solution of an obstacle problem for a class of PDIEs with a nonlinear Neumann boundary condition by introducing a class of generalized reflected BSDEs driven by a Lévy process. Very recently, [5] proved the existence and uniqueness of a solution for multidimensional BSDE driven by a multidimensional Lévy process.

Motivated by the above works, the purpose of the present paper is to consider ABSDEs driven by the Teugels martingales associated with a Lévy process. The existence and uniqueness of the solution to the ABSDEs is proved by means of the fixed point theorem. As a fundamental tool, the comparison theorem plays an important role in the theory and applications of BSDEs. We also proved a comparison theorem for ABSDEs. It is very important to notice that the conditions on the generator f needed for the comparison theorem for ABSDEs are stronger than those needed for the existence and uniqueness theorem.

The paper is organized as follows. In Section 2, we introduce some preliminaries. Section 3 is devoted to the proof of the existence and uniqueness of the solution to ABSDEs driven by Teugels martingales. In Section 4, we give the comparison theorem for the solutions of ABSDEs.

2. Preliminaries

For $t \geqslant 0$, let $\mathcal{F}_t$ denote the σ-algebra generated by the family of random variables $\{X_s, 0 \leqslant s \leqslant t\}$ augmented with the $\mathbb{P}$-null sets of F. Fix a time interval $[0,T]$ and set $L_T^2 = L^2(\Omega, \mathcal{F}_T, \mathbb{P})$. We will denote by $\mathcal{P}$ the predictable sub-σ-field of $\mathcal{F}_T \otimes \mathcal{B}_{[0,T]}$. We introduce some notations.

- $H^2_{\mathcal{F}}(0,T;\mathbb{R})$ denote the space of square-integrable and $(\mathcal{F}_t)$ progressively measurable processes $\phi = \{\phi_t, t \in [0,T]\}$ such that $\| \phi \|^2 = \mathbb{E}[\int_0^T |\phi_t|^2\, dt] < \infty$.
- $M^2_{\mathcal{F}}(0,T;\mathbb{R})$ will denote the subspace of H^2_T formed by predictable processes.
- l^2 be the space of real valued sequences $(x_n)_{n\geqslant 0}$ such that $\sum_{i=0}^{\infty} x_i^2 < \infty$, and $\|x\|^2_{l^2} = \sum_{i=0}^{\infty} |x_i|^2$.
- $H^2_{\mathcal{F}}(0,T;l^2)$ and $M^2_{\mathcal{F}}(0,T;l^2)$ are the corresponding spaces of l^2-valued processes equipped with the norm $\| \phi \|^2_{l^2} = \mathbb{E}[\int_0^T \sum_{i=1}^{\infty} |\phi_t^{(i)}|^2\, dt]$.

Let $X = \{X_t, t \geqslant 0\}$ be a Lévy process defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. It is well known that X_t has a characteristic function of the form

$$E^{i\theta X_t} = \exp\left[ia\theta t - \frac{1}{2}\sigma^2\theta^2 t + t\int_{\mathbb{R}} \left(e^{i\theta x} - 1 - i\theta x I_{\{|x|<1\}}\right)\nu(dx)\right]$$

where $a \in \mathbb{R}$, $\sigma^2 > 0$, and the Lévy measure ν is a measure defined in $\mathbb{R} \setminus 0$ and satisfies:

$$\int_{\mathbb{R}} (1 \wedge x^2)\nu(dx) < \infty,$$

$$\exists \varepsilon > 0, \quad \int_{(-\varepsilon,\varepsilon)^c} e^{\lambda|x|}\nu(dx) < \infty, \quad \text{and for some } \lambda > 0.$$

This implies that the random variables X_t have moments of all orders, i.e.

$$\int_{\mathbb{R}} |x|^i \nu(dx) < \infty, \quad \forall i \geqslant 2$$

and that the characteristic function $\mathbb{E}(e^{i\theta X_t})$ is analytic in a neighborhood of 0. Moreover, it will ensure the existence of the predictable representation (see below), which we will use in our proofs. We refer to [14] or [1] for a detailed account of Lévy processes.

Following [6,7], we define, for every $i = 1, 2, \ldots$, the so-called power-jump processes $\{X_t^{(i)}, t \geqslant 0\}$ and their compensated version $\{Y_t^{(i)} = X_t^{(i)} - \mathbb{E}[X_t^{(i)}], t \geqslant 0\}$, also called the Teugels martingales, as follows:

$$X_t^{(1)} = X_t, \qquad X_t^{(i)} = \sum_{0<s\leqslant t} (\Delta X_s)^i, \quad i = 2, 3, \ldots,$$

$$Y_t^{(i)} = X_t^{(i)} - \mathbb{E}\left[X_t^{(i)}\right] = X_t^{(i)} - t\mathbb{E}\left[X_1^{(i)}\right], \quad i \geqslant 1.$$