



Random attractors of the stochastic damped forced Ostrovsky equation

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ABSTRACT

The existence of a random attractor is established for the stochastic damped forced Ostrovsky equation with additive noise in $L^2(\mathbb{R})$. The main method is the combination of the energy equation, a suitable splitting of the solution, and the bilinear estimates in Bourgain spaces.

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1. Introduction

This paper is concerned with the asymptotic behavior of solutions to the following damped forced Ostrovsky equation with additive noise defined in the entire space $\mathbb{R}$:

$$du - (\beta u_{xxx} + \alpha D_x^{-1}u - (u^2)_x - \lambda u + f) dt = \sum_{i=1}^m h_i dw_i, \quad (1.1)$$

$$u(x, 0) = u_0(x), \quad x \in \mathbb{R}, t > 0, \quad (1.2)$$

where $D_x^{-1} = \mathcal{F}_x^{-1} \frac{1}{i\xi} \mathcal{F}_x$, α, β, λ are real constants with $\beta \neq 0, \alpha > 0, \lambda > 0$, f is time independent, h_i ($i = 1, 2, \dots, m$) are given functions defined on $\mathbb{R}$, and $\{w_i(t)\}_{i=1}^m$ are mutually independent two-sided Wiener processes on a probability space.

When $\lambda = 0, f = 0, h_i = 0$ ($1 \leq i \leq m$), the system in (1.1) becomes the standard Ostrovsky equation

$$u_t - \beta u_{xxx} - \alpha D_x^{-1}u + (u^2)_x = 0, \quad x \in \mathbb{R}, t > 0. \quad (1.3)$$

The Ostrovsky equation (1.3) was derived by Ostrovsky [26] as a model for the propagation of weakly nonlinear dispersive long surface and internal waves of small amplitude in a rotating fluid. Here free surface $u(t, x)$ has been rendered nondimensional with respect to the constant depth h of liquid and gravitational acceleration g and the parameter $\alpha = n^2/c_0 > 0$ measures the effect of rotation, where the wave speed $c_0 = \sqrt{gh}$, and n is the local Coriolis parameter. The parameter β determines the type of dispersion, i.e., $\beta < 0$ (negative-dispersion) for surface and internal waves in the ocean and surface waves in a shallow channel with an uneven bottom [4] and $\beta > 0$ (positive-dispersion) for capillary waves on the surface of liquid or for oblique magneto-acoustic waves in plasma [17,15]. Eq. (1.1) models the situation when nonlinearity, dispersion, dissipation rotation and stochastic effects are taken into account at the same time.

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The well-posedness of (1.3)–(1.2) has been studied in [18,21,22,25,30] and the references therein. The well-posedness of (1.1)–(1.2) without noise was obtained in [19] in Bourgain function spaces $\tilde{X}_{s,b}$ (see below for a precise definition of $\tilde{X}_{s,b}$) with $b > 1/2$. The Bourgain function spaces were introduced by J. Bourgain [6] for the well-posedness of the Korteweg–de Vries (KdV) equation. This method was developed by Kenig, Ponce and Vega [23] and Tao [28] to study the Cauchy problem for nonlinear dispersive wave equations. In this paper, the existence and uniqueness of solutions of the stochastic damped forced Ostrovsky equations (1.1)–(1.2) are studied in the above Bourgain spaces $\tilde{X}_{s,b}$. Roughly speaking, the index b represents the smoothness in time. However, the stochastic system (1.1)–(1.2) has the same time regularity as Brownian motion with $b < 1/2$. Hence, when trying to apply this method, the spaces $\tilde{X}_{s,b}$ with $b < 1/2$ have to be encountered. The well-posedness for the stochastic KdV equation and the stochastic Camassa–Holm equation in Bourgain spaces $X_{s,b}$ with $b < 1/2$ was studied in [9,13].

Another interesting thing is the long-term behavior for system (1.1)–(1.2). It is known that the long-term behavior of random systems is captured by a pullback random attractor, which was introduced in [11,14] as an extension of the attractors theory of deterministic systems in [20,27,29]. In the case of bounded domains, the existence of random attractors for stochastic PDEs has been studied extensively by many authors (see [1,2,5,8,10,14,16] and the references therein). When the domain is the entire space $\mathbb{R}^n$, the existence of random attractors was established recently in [3,7,31] and the references therein. The crucial idea for the proof is the asymptotic compactness and existence of bounded absorbing sets for these equations. And the asymptotic compactness is usually proved by a tail-estimate. In this paper, the random attractors for (1.1)–(1.2) are obtained by the same idea as above. Instead of a tail-estimate, the asymptotic compactness for (1.1)–(1.2) is checked by splitting the solutions into a decaying part plus a regular part as in [19,29].

Note that the phase function of the semigroup of (1.1) has non-zero points, which makes a difference from that of the linear KdV equation and the linear KP equation, and also makes the problem much more difficult. Therefore, the Fourier restriction operators

$$P^N h = \frac{1}{2\pi} \int_{|\xi| \geq N} e^{ix\xi} \hat{h}(\xi) d\xi, \quad P_N h = \frac{1}{2\pi} \int_{\varepsilon \leq |\xi| \leq N} e^{ix\xi} \hat{h}(\xi) d\xi, \quad \forall N \geq \varepsilon > 0,$$

are used to eliminate the singularity of the phase function and to split the solution in Section 4. For simplicity, denote $P_N h = \frac{1}{2\pi} \int_{|\xi| \leq N} e^{ix\xi} \hat{h}(\xi) d\xi$.

The paper is organized as follows. In Section 2, some fundamental results on the existence of a pullback random attractor for random dynamical systems are given. In Section 3, the well-posedness of the stochastic damped forced Ostrovsky equation is proved. In Section 4, we first prove that the solutions for the equation are bounded, and then split the solutions into two parts, one uniformly bounded in $\tilde{H}^3(\mathbb{R})$ and the other decaying in $\tilde{L}^2(\mathbb{R})$. These estimates are necessary for proving the existence of bounded absorbing sets and the asymptotic compactness of the equation. In the last section, the asymptotic compactness of the solution operator and then the existence of a pullback random attractor are proved.

2. Preliminaries

In this section, we recall the notion of a random dynamical system (RDS) and a random attractor. For more details, see [1,2,8,10,14].

Let $(\Omega, \mathcal{F}, P)$ be a probability space and $\{\theta_t : \Omega \rightarrow \Omega, t \in \mathbb{R}\}$ a family of measure preserving transformations such that $(t, \omega) \rightarrow \theta_t \omega$ is measurable, $\theta_0 = id$, and $\theta_{t+s} = \theta_t \theta_s$ for all $s, t \in \mathbb{R}$. The flow $(\theta_t)_{t \in \mathbb{R}}$ together with the corresponding probability space $(\Omega, \mathcal{F}, P)$ is called a measurable dynamical system.

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ -algebra $\mathcal{B}(X)$. Let $d(\cdot, \cdot)$ denotes the Hausdorff semi-distance in X given by $d(Y, Z) = \sup_{y \in Y} \inf_{z \in Z} \|y - z\|_X$ for any $Y \subseteq X$ and $Z \subseteq X$.

Definition 2.1. A RDS over $(\theta_t)_{t \in \mathbb{R}}$ is a measurable map $\psi : \mathbb{R}^+ \times \Omega \times X \rightarrow X, (t, \omega, x) \rightarrow \psi(t, \omega, x)$, such that $\psi(0, \omega, x) = x$ and ψ satisfies the cocycle property, i.e.,

$$\psi(t + s, \omega, x) = \psi(t, \theta_s \omega, \psi(s, \omega, x)),$$

for all $t, s \in \mathbb{R}^+, x \in X$ and all $\omega \in \Omega$. ψ is said to be a continuous RDS if P -a.e. $\omega \in \Omega, x \rightarrow \psi(t, \omega, x)$ is continuous for all $t \in \mathbb{R}^+$.

Hereafter, we always assume that ψ is a continuous RDS on X over $(\Omega, \mathcal{F}, P, (\theta_t)_{t \in \mathbb{R}})$.

Definition 2.2. (i) A set valued mapping $K : \Omega \rightarrow 2^X$ is said to be measurable if the map $\omega \rightarrow d(x, K(\omega))$ is measurable for each $x \in X$. A closed set valued measurable mapping is also called a random closed set.

(ii) Let $\mathcal{D}$ be a collection of random subsets of X and a random closed set $\{K(\omega)\}_{\omega \in \Omega} \in \mathcal{D}$. Then $\{K(\omega)\}_{\omega \in \Omega}$ is called a random absorbing set for ψ in $\mathcal{D}$ if for every $\{B(\omega)\}_{\omega \in \Omega} \in \mathcal{D}$ and P -a.e. $\omega \in \Omega$, there exists $t_B(\omega) > 0$ such that

$$\psi(t, \theta_{-t} \omega, B(\theta_{-t} \omega)) \subset K(\omega), \quad \text{for all } t \geq t_B(\omega).$$

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