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Parameter identification for weakly damped shallow arches

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The paper considers shallow arches under weak damping with hinged and clamped boundary conditions. A self-contained presentation of the existence, uniqueness, and regularity of the solutions is provided. The solutions are shown to be continuously dependent on the governing parameters. Furthermore, the solutions are shown to be weakly Gâteaux differentiable. The characterization of the weak Gâteaux derivative is established by introducing the notion of the weakened solution. Such solutions extend the class of weak solutions utilizing J. Lions' Method of Transposition.

The parameter identification problem is stated in terms of the best fit to data objective function. This function is shown to be Fréchet differentiable in the interior of the admissible set. The optimal parameters are characterized by a bang–bang control principle.

1. Introduction

Let a shallow arch be positioned over the interval $[0, L]$. Suppose that $y_0(x),\ x \in [0, L]$ is the arch shape when no load is applied to it. Let $y(x, t)$ be the deflection of the arch from its load-free shape y_0, when the arch is subjected to the dynamic load $q(x, t)$ for $x \in [0, L]$ and $t \geq 0$.

We consider the following governing equation for the arch dynamics

$$EI\frac{\partial^4 y}{\partial x^4} - \left[\frac{EA}{2L}\int_0^L \left\{\left(\frac{\partial y}{\partial x}\right)^2 + 2\frac{\partial y_0}{\partial x}\frac{\partial y}{\partial x}\right\} dx\right]\left(\frac{\partial^2 y_0}{\partial x^2} + \frac{\partial^2 y}{\partial x^2}\right) + c\frac{\partial y}{\partial t} + \rho A\frac{\partial^2 y}{\partial t^2} - q = 0, \tag{1.1}$$

where ρ is the mass density, A is the cross-section area of the arch, E is the Young's modulus, and c is the air damping coefficient, see [4,11,13]. Here I is the moment of inertia of the arch cross-section, and k is the radius of gyration.

The boundary conditions for y are either of the hinged type

$$y(0, t) = y(L, t) = \frac{\partial^2 y}{\partial x^2}(0, t) = \frac{\partial^2 y}{\partial x^2}(L, t) = 0, \quad t \geq 0, \tag{1.2}$$

or of the clamped type

$$y(0, t) = y(L, t) = \frac{\partial y}{\partial x}(0, t) = \frac{\partial y}{\partial x}(L, t) = 0, \quad t \geq 0. \tag{1.3}$$

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It is assumed that the shape y_0 also satisfies the boundary conditions.

The change of variables (using the same symbols to simplify the notation)

$$x \leftarrow \frac{\pi x}{L}, \qquad y_0 \leftarrow \frac{y_0}{k}, \qquad y \leftarrow \frac{y}{k}, \qquad p = \frac{q}{EIk}\left(\frac{L}{\pi}\right)^2, \qquad \omega_0 = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}},$$
$$t \leftarrow \omega_0 t, \qquad \kappa = \frac{c}{\rho A \omega_0}$$

in (1.1) leads to the dimensionless equation

$$\frac{\partial^2 y}{\partial t^2} + \frac{\partial^4 y}{\partial x^4} - \left[\frac{1}{2\pi}\int_0^\pi \left\{\left(\frac{\partial y}{\partial x}\right)^2 + 2\frac{\partial y_0}{\partial x}\frac{\partial y}{\partial x}\right\} dx\right]\left(\frac{\partial^2 y_0}{\partial x^2} + \frac{\partial^2 y}{\partial x^2}\right) + \kappa\frac{\partial y}{\partial t} - p = 0. \tag{1.4}$$

Let $y^* = y + y_0$, so y^* is the deflection of the arch from the x-axis. Make this change of variables in (1.4), and then rename y^* by y to get

$$\frac{\partial^2 y}{\partial t^2} + \frac{\partial^4 y}{\partial x^4} - \left[\beta + \frac{1}{2\pi}\int_0^\pi \left(\frac{\partial y}{\partial x}\right)^2 dx\right]\frac{\partial^2 y}{\partial x^2} + \kappa\frac{\partial y}{\partial t} = f, \tag{1.5}$$

where

$$\beta = -\frac{1}{2\pi}\int_0^\pi \left(\frac{\partial y_0}{\partial x}\right)^2 dx, \qquad f = \frac{\partial^4 y_0}{\partial x^4} + p.$$

The boundary conditions of the hinged or the clamped type have the same form as above, but now on the interval $[0, \pi]$.

This problem is supplemented with the initial conditions

$$y(x, 0) = u_0(x), \qquad \frac{\partial y}{\partial t}(x, 0) = v_0(x), \quad x \in [0, \pi]. \tag{1.6}$$

Eq. (1.5) is a particular case of the more general equation

$$y_{tt} + \alpha\Delta^2 y - \left(\beta + \gamma\int_\Omega |\nabla y|^2\, dx + \delta\left|\int_\Omega \nabla y \cdot \nabla y_t\, dx\right|^{q-2}\int_\Omega \nabla y \cdot \nabla y_t\, dx\right)\Delta y$$
$$+\xi y + \kappa y_t - \lambda\Delta y_t + \mu\Delta^2 y_t = f \tag{1.7}$$

considered in [5] on a bounded domain $\Omega \in \mathbb{R}^d$, with $q \in [2, \infty)$.

Comparing (1.5) with (1.7) we conclude that in our case the elasticity coefficient $\alpha = 1$, the extensibility coefficient $\gamma = 1/2\pi$, the Balakrishnan–Taylor damping coefficient $\delta = 0$, the viscous damping coefficient $\lambda = 0$, the strong damping coefficient $\mu = 0$, and $\xi = 0$. No sign conditions are imposed on the axial force coefficient β, and the weak damping coefficient κ. This means that we consider the weak damping case (i.e. $\delta = \lambda = \mu = 0$).

Equation

$$\frac{\partial^2 y}{\partial t^2} + \alpha\frac{\partial^4 y}{\partial x^4} - \left[\beta + \gamma\int_0^L \left(\frac{\partial y}{\partial x}\right)^2 dx\right]\frac{\partial^2 y}{\partial x^2} - \delta\int_0^L \frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial x\partial t}\, dx\,\frac{\partial^2 y}{\partial x^2} + \kappa\frac{\partial y}{\partial t} + \mu\frac{\partial^5 y}{\partial x^4\partial t} = 0 \tag{1.8}$$

was studied in [2]. The case of (1.8) with $\delta = \mu = \kappa = 0$ was considered in [1]. Paper [2] establishes the existence and the uniqueness of the weak solution for (1.8) for hinged and clamped boundary conditions. It also studies the behavior of its solutions as $t \to \infty$. Additional references on the behavior of solutions of (1.7) can be found in [5].

In this paper we study the weak damping problem ($\alpha > 0,\ \gamma > 0$)

$$y_{tt} + \alpha\Delta^2 y - \left[\beta + \gamma\int_\Omega |\nabla y|^2\, dx\right]\Delta y + \kappa y_t = f, \tag{1.9}$$

with $y(x, 0) = u_0(x),\ y_t(x, 0) = v_0(x),\ x \in \Omega \subset \mathbb{R}^d$. The boundary conditions are

$$y = \Delta y = 0, \quad (x, t) \in \partial\Omega \times (0, T)$$

for the hinged type, and

$$y = \frac{\partial y}{\partial n} = 0, \quad (x, t) \in \partial\Omega \times (0, T)$$

for the clamped type.