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Global existence and optimal decay rate for the strong solutions in H^2 to the 3-D compressible Navier–Stokes equations without heat conductivity ☆

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ABSTRACT

In this paper, we consider the 3-D compressible Navier–Stokes equations without heat conductivity, which form a hyperbolic–parabolic system. We prove the global existence of a strong solution when the initial perturbation is small in H^2 and its L^1-norm is bounded. Moreover, we can obtain the optimal decay rates for such a solution.

1. Introduction

The 3-D compressible Navier–Stokes equations for $(x, t) \in \mathbb{R}^3 \times \mathbb{R}_+$ are written as

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0, \\ \rho[u_t + (u \cdot \nabla)u] + \nabla p(\rho, \theta) = \mu \Delta u + (\mu + \mu')\nabla(\nabla \cdot u), \\ \rho c_V[\theta_t + (u \cdot \nabla)\theta] + \theta p_\theta(\rho, \theta)\nabla \cdot u = \kappa \Delta\theta + \Psi(u), \end{cases} \tag{1.1}$$

which govern the motion of gases, where ρ, u, θ represent the density, velocity and temperature of the fluid, respectively, κ is the coefficient of heat conduction and $p = p(\rho, \theta)$ is the pressure. In this paper, we study the case when the coefficient of heat conduction $\kappa = 0$. The viscosity coefficients $\mu > 0$ and μ' are the constants with $\mu' + \frac{2}{3}\mu \geq 0$, the specific heat $c_V > 0$ at constant volume is a constant, and $\Psi = \Psi(u)$ is the classical dissipation function:

$$\Psi(u) = \frac{\mu}{2}\sum_{i,j=1}^{3}(\partial_i u_j + \partial_j u_i)^2 + \mu' \sum_{j=1}^{3}(\partial_j u_j)^2.$$

The gas is ideal and polytropic, i.e., $p = \rho\theta$, $e = c_V\theta$, where e is the internal energy.

It is well known that all thermodynamics variables ρ, θ, e, p as well as the entropy s can be denoted by functions of any two of them. We take the two variables to be p and s. Then the equation of state for the gas is given by

$$\rho = ap^{\frac{c_V}{c_V+1}} \exp\left(-\frac{s}{c_V + 1}\right), \tag{1.2}$$

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where $a > 0$ is a constant. Under the aforementioned assumptions, system (1.1) in terms of the variables p, u and s reads

$$\begin{cases} p_t + \dfrac{1+c_V}{c_v} p\nabla \cdot u + u \cdot \nabla p = \dfrac{\Psi(u)}{c_V}, \\ u_t + (u \cdot \nabla)u + \dfrac{\nabla p}{\rho} = \dfrac{\mu}{\rho}\Delta u + \dfrac{\mu + \mu'}{\rho}\nabla(\nabla \cdot u), \\ s_t + (u \cdot \nabla)s = \dfrac{\Psi(u)}{p}, \end{cases} \tag{1.3}$$

where $\rho = \rho(p, s)$ is defined by (1.2). Notice that (1.3) is a hyperbolic–parabolic system, where the dissipation comes from viscosity. We consider the initial value problem to (1.3) in the whole space $\mathbb{R}^3$ with the initial data

$$(p, u, s)(0, x) = (p_0, u_0, s_0) \to (p_\infty, 0, s_\infty) \quad \text{as } |x| \to \infty, \tag{1.4}$$

where $p_\infty > 0$ and s_∞ are given constants.

There are a lot of mathematical results about the existence, stability and large-time behavior of the solutions to the compressible Navier–Stokes equations. Among them, when the coefficient of heat conduction $\kappa > 0$, Matsumura and Nishida [1–3], Ponce [4], Deckelnick [5,6], Hoff–Zumbrun [7,8], Liu–Wang [9] studied the problem. When there is an external potential force, the optimal decay rates were obtained in a series of paper [10–12]. When there is no heat conductivity, for the one-dimensional compressible Navier–Stokes equations in the Lagrangian coordinates, Liu–Zeng [13] obtained the elaborate pointwise estimates and large-time behavior of the solutions. Kawashima [14] proved the global existence and some references therein. In particular, Duan–Ma in [15] obtained the global existence and convergence rates of the classical solutions for the 3-D compressible Navier–Stokes equations without heat conductivity. However, all the previous decay rates were proved for the solutions in H^3 or more regular solutions. In this paper, we will refine the works [1] to show the global existence and optimal decay rates for the strong solutions to the initial value problem (1.3) and (1.4) in the H^2 framework when the initial data is a small perturbation of a constant state. Main results of this paper are stated as the following theorem.

Theorem 1.1. *Let p_∞, s_∞ be positive constants, there exists a constant ε_0 such that if $\|(p_0 - p_\infty, u_0, s_0 - s_\infty)\|_{H^2} \le \varepsilon_0$ and $\|(p_0 - p_\infty, u_0)\|_{L^1}$ is bounded. Then there exists a unique global solution (p, u, s) of the initial value problem (1.3)–(1.4) satisfying*

$$\|(p - p_\infty, u)(t)\|_{H^2}^2 + \int_0^t (\|\nabla p(\tau)\|_{H^1}^2 + \|\nabla u(\tau)\|_{H^2}^2) d\tau \le C\|(p_0 - p_\infty, u_0)\|_{H^2}^2, \tag{1.5}$$

$$\|(s - s_\infty)(t)\|_{H^2} \le C\|(p_0 - p_\infty, u_0, s_0 - s_\infty)\|_{H^2} \exp(C\|(p_0 - p_\infty, u_0)\|_{L^1 \cap H^2}). \tag{1.6}$$

Finally, there is a constant C_0 such that for any $t \ge 0$, the solution (p, u, s) has the decay properties

$$\|\nabla(p - p_\infty, u)(t)\|_{H^1} \le C_0(1 + t)^{-\frac{5}{4}}, \tag{1.7}$$

$$\|(p - p_\infty, u)(t)\|_{L^\infty} \le C_0(1 + t)^{-\frac{5}{4}}, \tag{1.8}$$

$$\|(p - p_\infty, u)(t)\|_{L^q} \le C_0(1 + t)^{-\frac{3}{2}(1-\frac{1}{q})}, \quad 2 \le q \le 6, \tag{1.9}$$

$$\|\partial_t(p, u, s)(t)\|_{L^2} \le C_0(1 + t)^{-\frac{5}{4}}. \tag{1.10}$$

Remark 1.2. The boundedness of $\|(p_0 - p_\infty, u_0)\|_{L^1}$ is used in the proof of the global existence. This is different from the previous work [3] when the coefficient of heat conduction $\kappa > 0$, where only H^3-norm of the perturbation is supposed for the global existence. In addition, due to lack of heat conductivity, the entropy s is nondissipative and thus has no decay-in-time property. However, all time derivatives $\partial_t(p, u, s)$ in L^2-norm decay in time.

The rest of this paper is devoted to prove Theorem 1.1 in Section 2. First, we will reformulate the problem. Second, we will get the optimal decay rate and global existence by closing the priori estimate.

In this paper, we use the standard notations L^p, H^s to denote the L^p and Sobolev spaces on $\mathbb{R}^3$, with norms $\|\cdot\|_{L^p}$ and $\|\cdot\|_{H^s}$ respectively. We use C to denote the constants depending only on physical coefficients and denote C_0 to be constants depending additionally on the initial data.

2. Proof of Theorem 1.1

In this section, we will first rewrite the system as follows. Set

$$\lambda = \sqrt{\frac{c_v}{(1 + c_V)\rho_\infty p_\infty}}, \qquad \lambda_1 = \sqrt{\frac{(1 + c_V)p_\infty}{c_V \rho_\infty}}, \qquad \bar{\mu} = \frac{\mu}{\rho_\infty}, \qquad \sigma = \frac{\mu + \mu'}{\rho_\infty},$$

where $\rho_\infty = \rho(p_\infty, s_\infty)$.