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The integral invariants of parallel timelike ruled surfaces

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ABSTRACT

In this paper, we first gave the parallel timelike ruled surface with a timelike ruling and its geometric invariants in terms of the main surface. We then obtained the integral invariants which are the pitch and the dual angle of pitch of the parallel timelike ruled surface with a timelike ruling and also the relations among them and integral invariants of the main surface.

1. Introduction

Ruled surfaces have been widely applied in designing cars, ships, manufacturing of products and many other areas such as motion analysis and simulation of rigid body and model-based object recognition systems. Modern surface modeling systems include ruled surfaces. The geometry of ruled surfaces is essential for studying kinematical and positional mechanisms in $\mathbb{R}^3$.

Dual numbers were introduced by Clifford as a tool for his geometrical investigations [1]. After him, Study used dual numbers and dual vectors in his research on the geometry of lines and kinematics [2]. In 1951, Müller used the real pitch $\mathcal{L}_{v_1}$ and the real pitch angle λ_{v_1} which are two real integral invariants in a study where he generalized the rigid body to v_1-closed ruled surface with the help of oriented v_1-line [3]. Skreiner in 1966 studied the geometry and kinematics of instantaneous spatial motion, using new geometric explanations he gave some theorems and results for the invariants of a closed ruled surface generated by an oriented line of a moving rigid body in $\mathbb{R}^3$ [4]. In 1972, Hacısalihoğlu generalized Steiner and Holditch theorems for the plane and the sphere to the line space in a paper about the pitch of a closed ruled surface [5]. Gürsoy presented some papers in 1990. In the paper titled "the dual angle of pitch of a closed ruled surface", he introduced a new dual integral invariant and generalized Holditch and Steiner theorems in the plane kinematics to the line space [6,7]. Later in another study he showed that the dual angle of pitch as the dual integral invariant of a closed ruled surface, corresponds to the dual spherical area described by the dual spherical indicatrix of the closed ruled surface. In 1994, Yapar found some relations among the integral invariants of the closed ruled surfaces which correspond to the dual closed curves in a dual closed strip [8]. In 1994, Güneş and Keleş examined the relation between the Steiner vector of a one-parameter closed spherical motion and the area vector of the closed spherical curve generated under the motion [9]. Then using the area formula given by Blaschke and the area vector defined by Müller, they obtained the formula given by Hacısalihoğlu in a different way [5]. Köse gave the pitch and the angle of pitch of the closed ruled surface in terms of the integral invariants of the dual spherical closed curve which corresponds to the closed ruled surface in 1997 [10]. Schaaf and Ravani in their paper (1998) developed a theory for higher order continuity of ruled surfaces constructed from ruled surface patches using dual

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curvature and torsion concepts [11]. In 1999, Gürsoy and Küçük gave some results on the geometric invariants, the pitch and the angle of pitch, of the closed trajectory ruled surfaces for spatial motions [12].

In recent times, ruled surfaces in Lorentz space have been studied (cf. [13–16]). Nizamoğlu studied parallel ruled surfaces in Euclidean space where he considers them as one-parameter dual curves on the dual unit sphere [17]. Uğurlu and Çalışkan proved that one-(real)parameter motions on the dual Lorentzian sphere and dual hyperbolic sphere correspond uniquely to spacelike and timelike ruled surfaces in three dimensional Lorentz space, respectively [18]. Özyılmaz and Yaylı examined the closed dual spherical motions on $\mathbb{D}_1^3$ in 2001 [14]. They gave the dual angle of pitch and real integral invariants of a closed timelike ruled surface. Then they obtained some relations between the dual Steiner vector of the closed dual spherical motion and the dual angle of pitch of the closed timelike ruled surfaces. In 1999, Şenyurt [19] obtained characteristic properties of the parallel ruled surfaces using the definition of parallel ruled surfaces corresponding to unit dual spherical parallel curves obtained with the help of dual angles given by Blaschke in 1930 [20]. In 2008, Çöken et al. wrote a paper on parallel timelike ruled surfaces with timelike rulings. They compared geometric invariants of the two parallel ruled surfaces. Considering the case that striction curves are null curves, they obtained the geodesic curvature, the normal curvature and the geodesic torsion of the curve [13].

In this paper, we first calculate pitch length and dual pitch angle of parallel timelike ruled surfaces with timelike ruling corresponding to dual unit Lorentzian spherical parallel timelike curves in lines space. Then, we find the relations among them and integral invariants of the main surface.

2. Preliminaries

For clarity and notation we recall some fundamental concepts of the subject.

Definition 1. A linear combination $a + \varepsilon\bar{a}$, where a and $\bar{a}$ are some real numbers and $\varepsilon^2 = 0$, is called a *dual number*.

The set of dual numbers, denoted by $\mathbb{D}$, is an associative ring with the unit element 1.

Definition 2. A *dual vector* is a triple of dual numbers and hence if $\vec{A}$ is a dual vector, we may write $\vec{A} = \vec{a} + \varepsilon\vec{\bar{a}}$, where $\vec{a}, \vec{\bar{a}} \in \mathbb{R}^3$ and ε is the dual unit as introduced above.

Henceforth, we will omit the arrows.

Definition 3. The set of all dual vectors is called the *dual space* and denoted by $\mathbb{D}^3$.

The dual inner-product may be imposed in a Lorentzian structure by

$$\langle A, B\rangle = \langle a, b\rangle + \varepsilon(\langle a, \bar{b}\rangle + \langle \bar{a}, b\rangle), \tag{1}$$

where $A = a+\varepsilon\bar{a}$ and $B = b+\varepsilon\bar{b}$ are some dual vectors and $\langle , \rangle$ is a Lorentz metric of signature $(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (-1, +1, +1)$. Then $(\mathbb{D}^3, \langle , \rangle)$ is denoted by $\mathbb{D}_1^3$. The causal character (cf. [17, p. 56]) of a dual vector $A = a + \varepsilon\bar{a}$ is defined by the causal character of its real part a, namely, A is spacelike, timelike or null if $\langle a, a\rangle$ is positive, negative or zero, respectively.

Definition 4. The Lorentzian cross-product on $\mathbb{D}_1^3$ is defined by

$$A \wedge B = a \wedge b + \varepsilon(a \wedge \bar{b} + \bar{a} \wedge b), \tag{2}$$

where $\wedge$ stands for the Lorentzian cross-product in $\mathbb{R}^3$ given by

$$a \wedge b = \sum_{i=1}^{3} \varepsilon_i \det(e_i, a, b)e_i. \tag{3}$$

For detailed information on the Lorentzian cross-product on $\mathbb{D}_1^3$, see [18].

3. Timelike ruled surfaces with timelike rulings

Uğurlu and Çalışkan [18] showed that a timelike (resp. spacelike) dual unit speed curve, depending on a real parameter in $\mathbb{D}_1^3$ represents uniquely a directed timelike (resp. spacelike) line in R_1^3. Veldkamp [21] and Study [2] use the term "dual point" to express a point of a dual curve. If every $r_1(t)$ and $\bar{r}_1(t)$ real valued functions are differentiable, the dual curve

$$\begin{aligned} R_1 :\quad & I \longrightarrow \mathbb{D}_1^3 \\ & t \longrightarrow R_1(t) = r_1(t) + \varepsilon\bar{r}_1(t) \end{aligned} \tag{4}$$

in D_1^3 is differentiable. By using the dual vector representation, the Plücker vectors r_1 and $p \times r_1$ of a timelike line L can be collected into a single dual timelike vector $R_1 = r_1 + \varepsilon\bar{r}_1$, where r_1 is the direction vector of L and p is the position vector of