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Existence and concentration of solutions for a class of biharmonic equations

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ABSTRACT

Some superlinear fourth order elliptic equations are considered. A family of solutions is proved to exist and to concentrate at a point in the limit. The proof relies on variational methods and makes use of a weak version of the Ambrosetti–Rabinowitz condition. The existence and concentration of solutions are related to a suitable truncated equation.

1. Introduction

In the last years, many authors have been studied several questions about the following Schrödinger elliptic equation

$$\begin{cases} -\epsilon^2 \Delta u + V(x)u = f(x,u) & \text{in } \Omega, \\ u \in H^1(\Omega) \end{cases} \tag{1.1}$$

with Neumann or Dirichlet boundary conditions, where Ω is a domain in $\mathbb{R}^N$. Motivated by Floer and Weinstein [12], Rabinowitz in [16] uses a mountain-pass type argument to find ground-state solutions to (1.1) for $\epsilon > 0$ sufficiently small, when $N \geqslant 3$, $\Omega = \mathbb{R}^N$, f is a subcritical and superlinear nonlinearity function and the potential V is nonnegative and assumed to satisfy the condition

$$0 < V_0 = \inf_{x \in \mathbb{R}^N} V(x) < \liminf_{|x| \to \infty} V(x). \tag{1.2}$$

In [20], Wang proves that the mountain-pass solutions found in [16] concentrate around a global minimum of V as $\epsilon \to 0$. In [2,3], Alves and Figueiredo consider the problem (1.1) for the p-Laplace operator obtaining existence, multiplicity and concentration of positive solutions. In the celebrated paper [10], del Pino and Felmer obtained existence and concentration of solutions for the problem (1.1), where $N \geqslant 3$, f is a subcritical and superlinear nonlinearity function and the potential V is nonnegative and it is assumed to satisfy the following condition

$$\inf_{x \in \Lambda} V(x) < \inf_{x \in \partial \Lambda} V(x),$$

where Λ is a bounded domain compactly contained in Ω. They developed a penalization-type method in order to overcome the lack of compactness and used the Mountain Pass Theorem to get existence and concentration of solutions. These

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arguments have inspired many authors in the last years, among them we could cite [4] and [11], where the authors have obtained multiplicity and concentration of nodal and positive solutions, respectively, to an equation related to (1.1). In [8], Alves and Soares obtain existence and concentration of nodal solutions of (1.1) for the case where $N = 2$ and the function f has critical exponential growth.

Although there are many works dealing with problem (1.1) and with related p-Laplacian ones, just a few works can be found dealing with biharmonic or even polyharmonic Schrödinger equations. Among them we could cite [5] and [6], where the authors have obtained nontrivial solutions to semilinear biharmonic problems with critical nonlinearities and also [18], where the authors obtained infinitely many solutions for a polyharmonic Schrödinger equation with non-homogeneous boundary data on unbounded domains.

Motivated by the results just described, a natural question is whether the same phenomenon of concentration occurs for the following class of fourth order elliptic equations

$$\begin{cases} \epsilon^4 \Delta^2 u + V(x)u = f(u) & \text{in } \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases} \tag{1.3}$$

where Δ^2 is the biharmonic operator, $\epsilon > 0$ and $N \geqslant 5$. The functions $f : \mathbb{R} \to \mathbb{R}$ and $V : \mathbb{R}^N \to \mathbb{R}$ satisfy the following assumptions:

(V_1) $V \in C^0(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$.
(V_2) There exist a bounded domain $\Omega \subset \mathbb{R}^N$ and $x_0 \in \Omega$, such that

$$0 < V(x_0) = V_0 = \inf_{\mathbb{R}^N} V < \inf_{\partial\Omega} V.$$

(f_1) $f \in C^1(\mathbb{R})$.
(f_2) $f(0) = f'(0) = 0$.
(f_3) There exist constants $c_1, c_2 > 0$ and $p \in (1, 2_* - 1)$, such that

$$|f(s)| \leqslant c_1|s| + c_2|s|^p, \quad \forall s \in \mathbb{R},$$

where $2_* = 2N/(N-4)$.
(f_4) $\lim_{|s|\to\infty} \frac{F(s)}{s^2} = +\infty$, where $F(s) = \int_0^s f(t)\,dt$.
(f_5) The function $f(s)/s$ is increasing for $s > 0$ and decreasing for $s < 0$.

Our main result is the following:

Theorem 1.1. *Assume that conditions (V_1), (V_2) and (f_1)–(f_5) hold. Then for each sequence $\epsilon_n \to 0$, there exists a subsequence, still denoted by $\{\epsilon_n\}$, such that, for all $n \in \mathbb{N}$, there exists a nontrivial weak solution u_n of (1.3) (with $\epsilon = \epsilon_n$). Moreover, if x_n is the maximum point of $|u_n|$, then $x_n \in \Omega$ and*

$$\lim_{n\to\infty} V(x_n) = \inf_{\mathbb{R}^N} V.$$

Although the principal arguments used here can be found in [10], the proofs have to be deeply modified because of some natural difficulties that the study of the biharmonic operator gives rise. For instance, in [10], in order to prove that the solutions of the penalized problem in fact are solutions of the original one, the authors use an argument that relies on the strong maximum principle to the Laplace operator and also on the fact that $u^+ = \max\{u, 0\}$ and $u^- = \min\{u, 0\}$ belong to $H^1(\Omega)$ for every u in $H^1(\Omega)$. In [7], to prove the same to a quasilinear problem, the authors combine a comparison principle together with Moser's iteration technique. However, all these arguments have severe limitations to deal with the biharmonic equation, because of the lack of a general form of the maximum principle to the biharmonic operator and the impossibility of splitting $u = u^+ + u^-$ in $H^2(\Omega)$. To overcome these problems, our argument relies on proving that rescalings of solutions of the penalized problem exhibit a uniform decay in infinity. To prove this we use some compactness results in Nehari manifolds found in [7] together with *a priori* L^p estimates found in [1] and L^∞ estimates proved by Ramos in [15].

It is worth to point out that we provide our results assuming a weaker version of the famous Ambrosetti–Rabinowitz condition (see (f_4)). The use of this weaker condition brings some difficulty to prove that the (*PS*) sequences are bounded, which required some arguments found in [13] (see also [17]). Moreover, this weak condition represents a difficulty to prove that the Nehari manifold is homeomorphic to the unitary sphere in $H^2(\mathbb{R}^N)$. This last problem can be dropped out using similar arguments of Szulkin and Weth in [19].

The article is organized in the following way: In the second section, we use the argument given by [10] and [13] to modify the function f to get the Palais–Smale condition for the functional associated with the respective modified equation. The existence and concentration of solutions to the modified problem are established. Finally, in the third section we prove that these solutions have a kind of uniform decay at infinity.