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Note

Blowup for the C^1 solutions of the Euler–Poisson equations of gaseous stars in R^N

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ABSTRACT

The Newtonian Euler–Poisson equations with attractive forces are the classical models for the evolution of gaseous stars and galaxies in astrophysics. In this paper, we use the integration method to study the blowup problem of the N-dimensional system with adiabatic exponent $\gamma > 1$, in radial symmetry. We could show that the C^1 non-trivial classical solutions (ρ, V), with compact support in $[0, R]$, where $R > 0$ is a positive constant with $\rho(t,r) = 0$ and $V(t,r) = 0$ for $r \geqslant R$, under the initial condition

$$H_0 = \int_0^R r^n V_0\,dr > \sqrt{\frac{2R^{2n-N+4}M}{n(n+1)(n-N+2)}} \tag{1}$$

with an arbitrary constant $n > \max(N-2, 0)$ and the total mass M, blow up before a finite time T for pressureless fluids or $\gamma > 1$. Our results could fill some gaps about the blowup phenomena to the classical C^1 solutions of that attractive system with pressure under the first boundary condition. In addition, the corresponding result for the repulsive systems is also provided. Here our result fully covers the previous case for $n = 1$ in [M.W. Yuen, Blowup for the Euler and Euler–Poisson equations with repulsive forces, Nonlinear Anal. 74 (2011) 1465–1470].

1. Introduction

The compressible isentropic Euler ($\delta = 0$) or Euler–Poisson ($\delta = \pm 1$) equations can be written in the following form:

$$\begin{cases} \rho_t + \nabla\cdot(\rho u) = 0, \\ \rho\big[u_t + (u\cdot\nabla)u\big] + \nabla P = \rho\nabla\Phi, \\ \Delta\Phi(t,x) = \delta\alpha(N)\rho \end{cases} \tag{2}$$

where $\alpha(N)$ is a constant related to the unit ball in R^N: $\alpha(1) = 1$; $\alpha(2) = 2\pi$ and for $N \geqslant 3$,

$$\alpha(N) = N(N-2)Vol(N) = N(N-2)\frac{\pi^{N/2}}{\Gamma(N/2+1)}, \tag{3}$$

where $Vol(N)$ is the volume of the unit ball in R^N and Γ is a Gamma function. As usual, $\rho = \rho(t,x) \geqslant 0$ and $u = u(t,x) \in \mathbf{R}^N$ are the density and the velocity respectively. $P = P(\rho)$ is the pressure function. The γ-law for the pressure term $P(\rho)$ could be applied:

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$$P(\rho) = K\rho^{\gamma} \tag{4}$$

which the constant $\gamma \geqslant 1$. If $K > 0$, we call the system with pressure; if $K = 0$, we call it pressureless.
When $\delta = -1$, the system is self-attractive. Eqs. (2) are the Newtonian descriptions of gaseous stars or a galaxy in astrophysics [2] and [4]. When $\delta = 1$, the system is the compressible Euler–Poisson equations with repulsive forces. It can be used as a semiconductor model [6]. For the compressible Euler equation with $\delta = 0$, it is a standard model in fluid mechanics [16]. And the Poisson equation $(2)_3$ could be solved by

$$\Phi(t,x) = \delta \int_{R^N} G(x-y)\rho(t,y)\,dy \tag{5}$$

where G is Green's function:

$$G(x) \doteq \begin{cases} |x|, & N = 1, \\ \log|x|, & N = 2, \\ \frac{-1}{|x|^{N-2}}, & N \geqslant 3. \end{cases} \tag{6}$$

Here, the solutions in radial symmetry could be:

$$\rho = \rho(t,r) \quad \text{and} \quad u = \frac{x}{r}V(t,r) =: \frac{x}{r}V \tag{7}$$

with the radius $r = (\sum_{i=1}^{N} x_i^2)^{1/2}$.
The Poisson equation $(2)_3$ becomes

$$r^{N-1}\Phi_{rr}(t,x) + (N-1)r^{N-2}\Phi_r = \alpha(N)\delta\rho r^{N-1}, \tag{8}$$

$$\Phi_r = \frac{\alpha(N)\delta}{r^{N-1}} \int_0^r \rho(t,s)s^{N-1}\,ds. \tag{9}$$

By standard computation, the systems in radial symmetry can be rewritten in the following form:

$$\begin{cases} \rho_t + V\rho_r + \rho V_r + \dfrac{N-1}{r}\rho V = 0, \\ \rho(V_t + VV_r) + P_r(\rho) = \rho\Phi_r(\rho). \end{cases} \tag{10}$$

In literature for constructing analytical solutions for these systems, interested readers could refer to [13,17,10,22,24]. The local existence for the systems can be found in [16,18,1,12]. The analysis of stabilities for the systems may be referred to [11,19–21,9,10,23,3,7,25].

In literature for showing blowup results for the solutions of these systems, Makino, Ukai and Kawashima firstly defined the tame solutions [19] for outside the compact of the solutions

$$V_t + VV_r = 0. \tag{11}$$

After this, Makino and Perthame continued the blowup studies of the "tame" solutions for the Euler system with gravitational forces [20]. Then Perthame proved the blowup results for 3-dimensional pressureless system with repulsive forces [21] ($\delta = 1$). In fact, all these results rely on the solutions with radial symmetry:

$$V_t + VV_r = \frac{\alpha(N)\delta}{r^{N-1}} \int_0^r \rho(t,s)s^{N-1}\,ds. \tag{12}$$

And the Emden ordinary differential equations were deduced on the boundary point of the solutions with compact support:

$$\frac{D^2R}{Dt^2} = \frac{\delta M}{R^{N-1}}, \qquad R(0,R_0) = R_0 \geqslant 0, \qquad \dot{R}(0,R_0) = 0 \tag{13}$$

where $\frac{dR}{dt} := V$ and M is the mass of the solutions, along the characteristic curve. They showed the blowup results for the C^1 solutions of the system (10).

In 2008 and 2009, Chae, Tadmor and Cheng in [3] and [7] showed the finite time blowup, for the pressureless Euler–Poisson equations with attractive forces ($\delta = -1$), under the initial condition,

$$S := \{a \in R^N \mid \rho_0(a) > 0,\ \Omega_0(a) = 0,\ \nabla \cdot u(0, x(0)) < 0\} \neq \phi \tag{14}$$

where Ω is the rescaled vorticity matrix $(\Omega_{0ij}) = \frac{1}{2}(\partial_i u_0^j - \partial_j u_0^i)$ with the notation $u = (u^1, u^2, \ldots, u^N)$ in their paper and some point x_0.