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Strong convergence of pairwise NQD random sequences

Rui Li [a,*], Weiguo Yang [b,1]

[a] *Department of Mathematics, Taiyuan Normal College, Taiyuan 030012, China*
[b] *Faculty of Science, Jiangsu University, Zhenjiang 212013, China*

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Abstract

Strong limit theory is one of the most important problems in probability theory. Some results on the convergence of pairwise NQD random sequences have been presented. This paper further analyzes the strong convergence of pairwise NQD sequences and generalizes partial results of Wu [Q.Y. Wu, Convergence properties of pairwise NQD random sequences, Acta Math. Sinica 45 (3) (2002) 617–624 (in Chinese)]. Since no general moment inequalities are given as so far, we avoid this problem and obtain a class of strong limit theorem for NQD sequences and some corresponding conclusions by use of truncation methods and generalized three series theorem, which are the supplements to the previous fruits.

Keywords: Pairwise NQD random variables; Strong convergence; Generalized three series theorem; Cr-inequality

1. Induction

Definition 1. The pair (X, Y) of random variables X and Y is said to be NQD (negatively quadrant dependent), if $\forall x, y \in R$, $P(X \leqslant x, Y \leqslant y) \leqslant P(X \leqslant x)P(Y \leqslant y)$. A sequence of random variables $\{X_n,\ n \geqslant 1\}$ is said to be pairwise NQD, if $\{X_i, X_j\}$ is NQD for every $i \neq j$, $i, j = 1, 2 \ldots.$

This definition given by Lehmann [3] in 1966 has been applied widely. Obviously, pairwise NQD sequence includes many negatively associated sequences, and pairwise independent random sequence is the most special case. Since strong limit theory is one of the most classical problems in probability theory, many scholars have dedicated themselves to the study of it. For example, Jardas [2] and Yang [8] studied the strong limit theorems of arbitrary random variables, Matula [4] gained the Kolmogorov-type strong law of large numbers for the identically distributed pairwise NQD sequences, Chen [1] discussed Kolmogorov–Chung strong law of large numbers for the non-identically distributed pairwise NQD sequences under very mild conditions, Wu [5] gave the three series theorem of pairwise NQD sequences and proved the Marcinkiewicz strong law of large numbers. The authors take the inspiration in [2,7]

* Corresponding author.
E-mail address: s0502062lirui@126.com (R. Li).
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and discuss the strong convergence of pairwise NQD sequences by applying truncation methods and generalized three series theorem, which extends partial results of Wu [5].

In this paper, we denote $X_n^a \triangleq -aI_{(X_n<-a)} + X_n I_{(|X_n|\leqslant a)} + aI_{(X_n>a)}$, $a>0$, and "$\ll$" as general "O."

Lemma 1. *(See [7].) Let $\{s_n, n\geqslant 1\}$ and $\{t_n, n\geqslant 1\}$ be non-negative number sequences, $s_n \leqslant t_n$ for each $n\geqslant 1$, then the infinite series*

$$\sum_{n=1}^{\infty}|u_n|^{s_n} < +\infty \quad \Longrightarrow \quad \sum_{n=1}^{\infty}|u_n|^{t_n} < +\infty.$$

Lemma 2. *(See [6].) Let X and Y be the NQD random variables, then*

(1) $E(XY) \leqslant EXEY$.
(2) $P(X>x, Y>y) \leqslant P(X>x)P(Y>y)$.
(3) *If $(r(\cdot), s(\cdot))$ is a non-decreasing (or non-increasing) function, then $r(X)$ and $s(Y)$ are still NQD random variables.*

Lemma 3 *(Generalized three series theorem). (See [6].) Let $\{X_n,\ n\geqslant 1\}$ be pairwise NQD sequences. For some $c>0$, we denote $X_n^c \triangleq -cI_{(X_n<-c)} + X_n I_{(|X_n|\leqslant c)} + cI_{(X_n>c)}$. If*

$$\sum_{n=1}^{\infty}P\big(|X_n|>c\big)<\infty, \qquad \sum_{n=1}^{\infty}EX_n^c<\infty, \qquad \sum_{n=1}^{\infty}\log^2 n \operatorname{Var} X_n^c<\infty,$$

then

$$\sum_{n=1}^{\infty}X_n \text{ converges a.s.}$$

2. Mainstream

Theorem 1. *Let $\{X_n,\ n\geqslant 1\}$ be a sequence of pairwise NQD random variables (r.v.s) and $EX_n=0$. Let $\{a_n,\ n\geqslant 1\}$ be a positive number sequence such that $a_n \uparrow \infty$. Suppose that $\varphi_n : R_+ \mapsto R_+$ be Borel functions and $\alpha_n \geqslant 1$, $\beta_n \leqslant 2$, $K_n \geqslant 1$, $M_n \geqslant 1$ $(n\in N)$ be constants satisfying*

$$0<x_1\leqslant x_2 \quad \Rightarrow \quad \frac{\varphi_n(x_1)}{x_1^{\alpha_n}} \leqslant K_n \frac{\varphi_n(x_2)}{x_2^{\alpha_n}}$$

and

$$\frac{x_1^{\beta_n}}{\varphi_n(x_1)} \leqslant M_n \frac{x_2^{\beta_n}}{\varphi_n(x_2)}. \tag{1}$$

If

$$\sum_{n=1}^{\infty}K_n \log^2 n \frac{E\varphi_n(|X_n|)}{\varphi_n(a_n)}<\infty, \tag{2}$$

$$\sum_{n=1}^{\infty}M_n \log^2 n \frac{E\varphi_n(|X_n|)}{\varphi_n(a_n)}<\infty, \tag{3}$$

then

$$\lim_{n\to\infty}\frac{1}{a_n}\sum_{k=1}^{n}X_k=0 \quad a.s. \tag{4}$$