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Stability of Timoshenko systems with past history ☆

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Abstract

We consider vibrating systems of Timoshenko type with past history acting only in one equation. We show that the dissipation given by the history term is strong enough to produce exponential stability if and only if the equations have the same wave speeds. Otherwise the corresponding system does not decay exponentially as time goes to infinity. In the case that the wave speeds of the equations are different, which is more realistic from the physical point of view, we show that the solution decays polynomially to zero, with rates that can be improved depending on the regularity of the initial data.

Keywords: Timoshenko system; Exponential stability; Polynomial rate of decay

1. Introduction

We consider the following linear Timoshenko system with past history:

$$\rho_1\varphi_{tt} - k(\varphi_x + \psi)_x = 0 \quad \text{in } (0, L) \times (0, \infty), \tag{1.1}$$

$$\rho_2\psi_{tt} - b\psi_{xx} + \int_0^\infty g(s)\psi_{xx}(x, t - s)\,ds + k(\varphi_x + \psi) = 0 \quad \text{in } (0, L) \times (0, \infty), \tag{1.2}$$

and initial conditions

$$\varphi(\cdot, 0) = \varphi_0, \quad \varphi_t(\cdot, 0) = \varphi_1, \quad \psi(\cdot, 0) = \psi_0, \quad \psi_t(\cdot, 0) = \psi_1 \quad \text{in } (0, L) \tag{1.3}$$

with positive constants ρ_1, ρ_2, k, b.

Here we are interested in the asymptotic behavior of the solutions. Our main tools are Prüss' results on the exponential stability of semigroups, see [7,8]. In order to use these results, it is necessary to embed the problem into

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the context of semigroup, so some modifications in our original system (1.1)–(1.2) should be made. We introduce the notation

$$\eta^t(x,s) := \psi(x,t) - \psi(x,t-s), \tag{1.4}$$

then system (1.1)–(1.2) is rewritten as

$$\rho_1\varphi_{tt} - k(\varphi_x + \psi)_x = 0, \tag{1.5}$$

$$\rho_2\psi_{tt} - \left(b - \int_0^\infty g(s)\,ds\right)\psi_{xx} - \int_0^\infty g(s)\eta^t_{xx}(x,s)\,ds + k(\varphi_x + \psi) = 0, \tag{1.6}$$

$$\eta_t + \eta_s - \psi_t = 0, \tag{1.7}$$

where the third equation is obtained differentiating (1.4) with respect to s. The initial conditions are given by

$$\varphi(\cdot,0) = \varphi_0, \quad \varphi_t(\cdot,0) = \varphi_1, \quad \psi(\cdot,0) = \psi_0, \quad \psi_t(\cdot,0) = \psi_1 \quad \text{in } (0,L), \tag{1.8}$$

$$\eta_0(\cdot,s) = \psi_0(\cdot,0) - \psi_0(\cdot,-s) \quad \text{in } (0,L)\times(0,\infty), \tag{1.9}$$

which means that the history is considered as an initial value. We consider Dirichlet boundary conditions, but our arguments can be used to prove similar results for other boundary conditions. Concerning the kernel g we consider the following hypotheses:

$$g(t) > 0, \quad \exists k_0, k_1, k_2 > 0\colon \quad -k_0 g(t) \leqslant g'(t) \leqslant -k_1 g(t), \quad \left|g''(t)\right| \leqslant k_2 g(t), \quad \forall t \geqslant 0, \tag{1.10}$$

$$\tilde{b} := b - \int_0^\infty g(s)\,ds > 0. \tag{1.11}$$

Let us mention some energy decay results for dissipative Timoshenko systems. In [2], Kim and Renardy considered a conservative Timoshenko system with two boundary feedbacks and they proved exponential stability for the energy associated to the system. If the history term in (1.2) is replaced by the control function $\bar{b}(x)\psi_t$, $\bar{b} > 0$, then Soufyane [9] proved exponential stability for the linearized system if and only if the waves speed of Eqs. (1.1), (1.2) are equal, that is,

$$\frac{\rho_1}{k} = \frac{\rho_2}{b}. \tag{1.12}$$

Similar results are obtained by Rivera and Racke [5], where semigroups techniques are used. In [4] the same authors consider a dissipative Timoshenko system with a dissipation through a coupling to a heat equation, and they show exponential stability if and only if (1.12) holds.

In [1], Ammar Khodja et al. consider also a Timoshenko system with memory effect but considering null history, in that case the system is called a Volterra integro-differential system. For the Volterra problem they proved the exponential stability provided the wave speeds are equal. When the wave speeds are different, the authors consider a class of kernels for which there is no exponential stability. No information is given concerning the decay in this case.

Introducing non-zero history on ψ makes the problem different from that considered in [1], so different techniques have to be used. The main result of this paper is to show that the system is exponentially stable if and only if the wave speeds of Eqs. (1.1), (1.2) are equal, that is, if and only if (1.12) holds. Moreover, the class of kernel that we consider here to prove the lack of exponential stability is larger than that considered in [1]. In particular our result implies the non-exponential stability for singular kernels. When the identity (1.12) does not hold, which is more interesting from the physical point of view, we show that the first-order energy decays polynomially with rates that depend on the regularity of the initial data.

The paper is organized as follows. In Section 2 we establish the existence and uniqueness results to system (1.5)–(1.7). The exponential stability of the semigroup associated to this system is studied in Section 3. In Section 4 we show the non-exponential stability of the semigroup. Finally, in Section 5 we show the polynomial decay when the wave speeds are different.