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Interpolation in weighted spaces of entire functions in $\mathbb{C}^2$

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Abstract

The purpose of this article is to construct complete interpolating sequences for special spaces of entire functions of two variables. The origin for the work is a result of Yu. Lyubarskii and A. Rashkovskii on sampling and interpolation for two-dimensional Fourier transforms. We also prove a theorem of Paley–Wiener type.

Keywords: Complete interpolating sequences; Paley–Wiener spaces

1. Introduction

Let M be the algebraic sum of a finite number of nonzero vectors in $\mathbb{C}^2$ (for the definition, see Section 3), and let

$$H_M(z) = \sup_{\lambda \in M} \operatorname{Re}\langle z, \lambda\rangle$$

be its support function. $\langle\cdot,\cdot\rangle$ denotes the $\mathbb{C}^2$-scalar product: $\langle z, \lambda\rangle = z_1\bar{\lambda}_1 + z_2\bar{\lambda}_2$ for $z = (z_1, z_2)$, $\lambda = (\lambda_1, \lambda_2) \in \mathbb{C}^2$. The geometrical interpretation of $H_M(z)$ is that if $|z| = 1$, then $H_M(z)$ is the length of the set M projected on the ray with direction z. In this article we study interpolation

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problems in the space $\mathcal{S}_M^p$, $1<p<\infty$, consisting of entire functions of exponential type[1] with indicator not exceeding $H_M(z)$, and with finite norm

$$\|f\|_{\mathcal{S}_M^p}=\sup_{\gamma\in\mathbb{C}^2}\left(\int\limits_{\mathbb{C}^2}\left|f(z)\right|^p e^{-pH_M(z)}\left(dd^c H_M(z+\gamma)\right)^2\right)^{1/p}<\infty. \tag{1.1}$$

The form $(dd^c H_M(z))^2$ is the Monge–Ampère measure of $H_M(z)$, and its definition is given in Section 3.

We say that a sequence $\Omega=\{\omega\}\subset\mathbb{C}^2$ is an interpolating sequence for $\mathcal{S}_M^p$ if for each $\{a_\omega e^{-H_M(\omega)}\}_{\omega\in\Omega}\in l^p(\Omega)$, there exists $f\in\mathcal{S}_M^p$ solving the interpolating problem

$$f(\omega)=a_\omega,\quad \omega\in\Omega. \tag{1.2}$$

The factor $e^{-H_M(\omega)}$ in $\{a_\omega e^{-H_M(\omega)}\}_{\omega\in\Omega}$ is introduced in order to compensate the exponential growth of f. If the solution to this problem is always unique we say that Ω is a *complete interpolating sequence* for $\mathcal{S}_M$. It will be shown that for each interpolating sequence Ω, the operator

$$f\mapsto\left\{f(\omega)e^{-H_M(\omega)}\right\}_{\omega\in\Omega}$$

is bounded from $\mathcal{S}_M^p$ onto l^p. So by the Banach inverse operator theorem each complete interpolating sequence Ω is also a *sampling* sequence, i.e.

$$A\left\|\left\{f(\omega)e^{-H_M(\omega)}\right\}\right\|_{l^p}\leqslant\|f\|_{\mathcal{S}_M^p}\leqslant B\left\|\left\{f(\omega)e^{-H_M(\omega)}\right\}\right\|_{l^p},\quad f\in\mathcal{S}_M^p,$$

for some $A,B>0$ independent of f.

The purpose of this article is to construct complete interpolating sequences for the space $\mathcal{S}_M^p$. In contrast to the one-dimensional case where a full description of complete interpolating sequences is obtained for a number of spaces of entire functions (see e.g. [13,14] and [11]), the corresponding multi-dimensional problem is very far from being solved. This is the case because the machinery of generating functions, the main tool in the one-dimensional case, cannot be applied in several dimensions. Even the existence of complete interpolating sequences is not obvious, so at the present stage it is natural to just look for examples of complete interpolating sequences in a hope to get a hint about more general situations.

Except natural constructions related to products of one-dimensional interpolating sequences, the only examples of complete interpolating sequences in several variables known to the author concern the spaces

$$PW_M=\left\{f(z)\colon\ f(z)=\int\limits_M e^{i\langle z,\xi\rangle}\phi(\xi)\,dm_\xi,\ \phi\in L^2(M)\right\}$$

endowed with the $L^2(\mathbb{R}^2)$-norm. Here $M\subset\mathbb{R}^2$ is a convex polygon, $\mathbb{R}^2$ is considered as the real plane in $\mathbb{C}^2$ and dm stands for the plane Lebesgue measure. This was studied in [10], where it was shown that the collection of all pairwise intersections of the zero hyperplanes of a certain function forms a complete interpolating sequence if it is uniformly separated (i.e. the distance between each pair of distinct points is uniformly bounded off zero).

Following the construction in [10] we construct entire functions with plane zeros in $\mathbb{C}^2$ which will generate complete interpolating sequences for $\mathcal{S}_M^p$. Being of complex dimension 1, the zero

[1] We refer the reader to [7] for main definitions and properties of entire functions of exponential type.