Available online at www.sciencedirect.com
ScienceDirect
J. Math. Anal. Appl. 334 (2007) 847–858
Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS
www.elsevier.com/locate/jmaa

Oscillation criteria for a class of second-order Emden–Fowler delay dynamic equations on time scales ☆

Zhenlai Han [a,b,*], Shurong Sun [a], Bao Shi [b]

[a] School of Science, Jinan University, Jinan, Shandong 250022, PR China
[b] Institute of Applied Mathematics, Naval Aeronautical Engineering Institute, Yantai, Shandong 264001, PR China

Received 8 October 2006
Available online 8 January 2007
Submitted by J.S.W. Wong

Abstract

By means of Riccati transformation technique, we establish some new oscillation criteria for the second-order Emden–Fowler delay dynamic equations

$$x^{\Delta\Delta}(t)+p(t)x^{\gamma}\big(\tau(t)\big)=0$$

on a time scale $\mathbb{T}$; here γ is a quotient of odd positive integers with $p(t)$ real-valued positive rd-continuous functions defined on $\mathbb{T}$. To the best of our knowledge nothing is known regarding the qualitative behavior of these equations on time scales. Our results in this paper not only extend the results given in [R.P. Agarwal, M. Bohner, S.H. Saker, Oscillation of second-order delay dynamic equations, Can. Appl. Math. Q. 13 (1) (2005) 1–18] but also unify the oscillation of the second-order Emden–Fowler delay differential equation and the second-order Emden–Fowler delay difference equation.

Keywords: Oscillation; Second-order; Delay dynamic equations; Time scale

☆ This research is supported by the Natural Science Foundation of China (10471077) and supported by Shandong Research Funds for Young Scientists (03BS094), also supported by National Science Foundation of Educational Department of Shandong Province (03P51) (J04A60) and Jinan University Research Funds for Doctors (B0621).
* Corresponding author.
E-mail addresses: hanzhenlai@163.com (Z. Han), sshrong@163.com (S. Sun), baoshi781@sohu.com (B. Shi).

0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2007.01.004

1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger in his PhD thesis in 1988 in order to unify continuous and discrete analysis (see Hilger [17]). Several authors have expounded on various aspects of this new theory; see the survey paper by Agarwal et al. [1] and references cited therein. A book on the subject of time scale, by Bohner and Peterson [9] summarizes and organizes much of the time scale calculus, we refer also the last book by Bohner and Peterson [8] for advances in dynamic equations on time scales. For the notions used below we refer to the next section that provides some basic facts on time scales extracted from Bohner and Peterson [9].

A time scale $\mathbb{T}$ is an arbitrary closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. Many other interesting time scales exist, and they give rise to plenty of applications, among them the study of population dynamic models which are discrete in season (and may follow a difference scheme with variable step-size or often modeled by continuous dynamic systems), die out, say in winter, while their eggs are incubating or dormant, and then in season again, hatching gives rise to a nonoverlapping population (see Bohner and Peterson [9]).

In recent years, there has been much research activity concerning the oscillation and nonoscillation of solutions of various equations on time scales, and we refer the reader to Akin-Bohner et al. [5–7], Bohner and Saker [10], Erbe [14], Erbe et al. [15], Saker [20,21]. However, there are few results dealing with the oscillation of the solutions of delay dynamic equations on time scales [2,3,11,19,22,24,25].

Agarwal et al. [2] considered the second-order delay dynamic equations on time scales

$$x^{\Delta\Delta}(t)+p(t)x\big(\tau(t)\big)=0 \quad \text{for } t\in\mathbb{T}, \tag{1.1}$$

and established some sufficient conditions for oscillation of (1.1).

To the best of our knowledge, there are no results regarding the oscillation of the solutions of the following second-order nonlinear delay dynamic equations on time scales up to now

$$x^{\Delta\Delta}(t)+p(t)x^{\gamma}\big(\tau(t)\big)=0 \quad \text{for } t\in\mathbb{T}. \tag{1.2}$$

Clearly, (1.1) is the special cases of (1.2). To develop the qualitative theory of delay dynamic equations on time scales, in this paper, we consider the second-order nonlinear delay dynamic equation on time scales (1.2).

As we are interested in oscillatory behavior, we assume throughout this paper that the given time scale $\mathbb{T}$ is unbounded above, i.e., it is a time scale interval of the form $[a,\infty)$ with $a\in\mathbb{T}$.

We assume that γ is a quotient of odd positive integer, $p(t)$ is positive, real-valued rd-continuous functions defined on $\mathbb{T}$, $\tau(t):\mathbb{T}\to\mathbb{T}$ is an rd-continuous function such that $\tau(t)\leqslant t$ and $\tau(t)\to\infty$ $(t\to\infty)$.

By a solution of (1.2), we mean a nontrivial real-valued function x satisfying (1.2) for $t\geqslant t_x\geqslant a$. A solution x of (1.2) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise it is called nonoscillatory. (1.2) is called oscillatory if all solutions are oscillatory. Our attention is restricted to those solutions x of (1.2) which exist on some half line $[t_x,\infty)$ with $\sup\{|x(t)|:\ t\geqslant t_0\}>0$ for any $t_0\geqslant t_x$.

We note that if $\mathbb{T}=\mathbb{R}$, then $\sigma(t)=0$, $\mu(t)=0$, $x^{\Delta}(t)=x'(t)$ and (1.2) becomes the second-order Emden–Fowler delay differential equation

$$x''(t)+p(t)x^{\gamma}\big(\tau(t)\big)=0 \quad \text{for } t\in\mathbb{R}. \tag{1.3}$$