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Stability criteria for certain high even order delay differential equations

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Abstract

In this paper we study the asymptotic stability of the zero solution of even order linear delay differential equations of the form

$$y^{(2m)}(t) = \sum_{j=0}^{2m-1} a_j y^{(j)}(t) + \sum_{j=0}^{2m-1} b_j y^{(j)}(t-\tau),$$

where a_j and b_j are certain constants and $m \geqslant 1$. Here $\tau > 0$ is a constant delay. In proving our results we make use of Pontryagin's theory for quasi-polynomials.

Keywords: Asymptotic stability; Stability criteria; Sufficient conditions; Delay; Characteristic functions; Stability regions

1. Introduction

The aim of this paper is to study the asymptotic stability of the zero solution of the delay differential equation

$$y^{(2m)}(t) = \sum_{j=0}^{2m-1} a_j y^{(j)}(t) + \sum_{j=0}^{2m-1} b_j y^{(j)}(t-\tau), \tag{1.1}$$

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where $\tau > 0$, a_j and b_j are constants and $m \geqslant 1$. In previous papers [1,2], we considered Eq. (1.1) with $m = 1$. See the text [3] for stability of second order delay equations and applications. Odd higher order equations were considered in [4]. The current paper and [4] will provide a complete treatment for higher order delay differential equations with constant coefficients and constant delay. There are no practical stability robust criteria of the zero solution of (1.1) for $m \geqslant 2$. For stability and oscillation of certain fourth order equations see [5–12]. See [13–16] for studies of systems that may shed light on (1.1). The study on systems does not, however, yield complete practical stability criteria of (1.1). It is clear that with $4m$ independent parameters in (1.1) one cannot expect to get regions of stability. Our goal is to derive algorithmic type stability criteria.

We take the view that part of the jth derivative term of the equation

$$y^{(2m)}(t) = \sum_{j=0}^{2m-1} p_j y^{(j)}(t) \tag{1.2}$$

is delayed and the remaining part is not. Note that with $\tau = 0$ the zero solution of (1.1) or (1.2) is asymptotically stable if and only if all the characteristic roots of a real polynomial

$$x^{2m} - p_{2m-1}x^{2m-1} - p_{2m-2}x^{2m-2} - p_{2m-3}x^{2m-3} - \cdots - p_0 = 0 \tag{1.3}$$

are in complex left half plane. Relative to (1.1), our view is that

$$p_j = a_j + b_j, \quad j = 0, 1, \ldots, 2m - 1. \tag{1.4}$$

By Hurwitz Criterion [17] all roots have negative real parts if and only if

$$\delta_j > 0, \quad j = 1, 2, \ldots, 2m, \tag{1.5}$$

where the δ_j are the following determinants:

$$\delta_1 = -p_{2m-1},$$

$$\delta_2 = \begin{vmatrix} -p_{2m-1} & -p_{2m-3} \\ 1 & -p_{2m-2} \end{vmatrix},$$

$$\delta_k = \begin{vmatrix} -p_{2m-1} & -p_{2m-3} & -p_{2m-5} & \cdots & -p_{2m-2k+1} \\ 1 & -p_{2m-2} & -p_{2m-4} & \cdots & -p_{2m-2k+2} \\ 0 & -p_{2m-1} & -p_{2m-3} & \cdots & -p_{2(m-k)+3} \\ 0 & 1 & -p_{2m-2} & \cdots & -p_{2(m-k)+4} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -p_{2m-k} \end{vmatrix}, \quad k = 3, \ldots, 2m,$$

with $-p_{2m-j} = 0$ for $j > 2m + 1$. For delay equations, there is no simple criterion as the Routh–Hurwitz criterion.

This paper is organized as follows. In Section 2, we present the tools used in our asymptotic stability analysis. In Section 3 we give our main results and some special cases. In Section 4 we present some examples.

Some of the results and techniques in this paper are different than the results of the odd case [4]. We have also sharpened stopping criteria of all algorithms.

2. Background

In this section, we identify the characteristic function of (1.1) in order to study the asymptotic stability of the zero solution. We also cite the main results of Pontryagin related to asymptotic stability [18] and the applications of Pontryagin's results [19, §13.7–13.9].