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Qualitative analysis of a ratio-dependent Holling–Tanner model

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Abstract

In this paper, we consider a Holling–Tanner system with ratio-dependence. First, we establish the sufficient conditions for the global stability of positive equilibrium by constructing Lyapunov function. Second, through a simple change of variables, we transform the ratio-dependent Holling–Tanner model into a better studied Liénard equation. As a result, the uniqueness of limit cycle can be solved.

Keywords: Global asymptotic stability; Limit cycle; Holling–Tanner model; Ratio-dependent; Lyapunov function

1. Introduction

Recently, the Leslie predator–prey model has received some interest, see [1–5]. Generally, the Leslie predator–prey model takes the form of

$$\begin{cases} \dfrac{dx}{dt} = xg(x) - p(x)y, \\ \dfrac{dy}{dt} = y\left[s\left(1 - \dfrac{y}{K(x)}\right)\right], \\ x(0) > 0, \quad y(0) > 0. \end{cases} \tag{1.1}$$

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In system (1.1), x and y stand for prey and predator population (or densities), respectively, as functions of time. The predator growth equation is of logistic type; s is the intrinsic growth rates of predator, but carrying capacity $K(x)$ is the function on the population size of prey.

It is assumed that the carrying capacity of predator's environment is proportional to prey abundance, i.e. $K = x/h$ (h is the conversion factor of prey into predators). We obtain the following model:

$$\begin{cases} \dfrac{dx}{dt} = xg(x) - p(x)y, \\ \dfrac{dy}{dt} = y\left[s\left(1 - h\dfrac{y}{x}\right)\right], \\ x(0) > 0, \quad y(0) > 0. \end{cases} \tag{1.2}$$

The form of the predator equation in system (1.2) was first introduced by Leslie [6]. The term hy/x of this equation is called the Leslie–Gower term. This interesting formulation for the predator dynamics has been discussed by Leslie and Gower in [7] and by Pielou in [8].

It is assumed that the prey grows logistically with growth rate r and carrying capacity k in the absence of predator, i.e. $g(x) = r(1 - \frac{x}{k})$. From system (1.2), we obtain the following model:

$$\begin{cases} \dfrac{dx}{dt} = rx\left(1 - \dfrac{x}{k}\right) - p(x)y, \\ \dfrac{dy}{dt} = y\left[s\left(1 - h\dfrac{y}{x}\right)\right], \\ x(0) > 0, \quad y(0) > 0. \end{cases} \tag{1.3}$$

$p(x)$ is predator function response to prey. The function $p(x) = \frac{my}{A+x}$ is referred to as a functional response of Holling type II; the parameter m is the maximal predator per capita consumption rate; the parameter A is the number of prey necessary to achieve one-half of the maximum rate m. For the derivation of the type II function response one can refer to [9]. According to Hassel and May [10], type II function response is the most common type of function response among arthropod predators. The Leslie predator–prey model with Holling type II function response is the following predator–prey model, which is called Holling–Tanner model

$$\begin{cases} \dfrac{dx}{dt} = rx\left(1 - \dfrac{x}{k}\right) - \dfrac{mx}{A+x}y, \\ \dfrac{dy}{dt} = y\left[s\left(1 - h\dfrac{y}{x}\right)\right], \\ x(0) > 0, \quad y(0) > 0. \end{cases} \tag{1.4}$$

S.B. Hsu and T.W. Hwang [2] derived the criterion for the locally asymptotical stability of the positive equilibrium of system (1.4) and they gave the conditions ($a \geqslant 1$ or $a < 1$ and $a + \delta \geqslant 1$, or $a + \delta < 1$ and $(1 - a - \delta)^2 - 8a\delta \leqslant 0$, where $a = A/k$, $\delta = s/r$) under which local stability of a positive equilibrium point implies global stability by the application of the Dulac criterion and the construction of Lyapunov function. However, the authors were unable to show that the system (1.4) has a unique limit cycle when the positive equilibrium exists and becomes unstable. E. Saez and E. Gonzalez-Olivares [3] described the bifurcation diagram of limit cycle that appears in the first realistic quadrant. The authors showed that local stability and global stability of the unique positive equilibrium are not equivalent for system (1.4). A. Gasull, R.E. Kooij and J. Torregrosa [4] also showed that the asymptotic stability of the positive equilibrium does