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ϕ_0-Stability of hybrid impulsive dynamic systems on time scales ☆

Peiguang Wang [a,*], Xia Liu [b,1]

[a] *College of Electronic and Information Engineering, Hebei University, Baoding 071002, China*
[b] *College of Mathematics and Computer Science, Hebei University, Baoding 071002, China*

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Abstract

In this paper, we shall employ the method of cone-valued Lyapunov functions and comparison principle to investigate the ϕ_0-stability of impulsive hybrid systems on time scales.

Keywords: Hybrid impulsive system; ϕ_0-Stability; Uniform ϕ_0-stability; Asymptotical ϕ_0-stability; Uniformly asymptotical ϕ_0-stability; Time scales

1. Introduction

Since there is a striking similarity or even duality between the theories of continuous and discrete dynamic systems, many results in the theory of difference equations have been obtained as more or less natural discrete analogs of corresponding results of differential equations. From a modelling point of view, it is more realistic to model a phenomena by a dynamic system that incorporates both continuous and discrete times, namely, time as an arbitrary closed set of reals known as time scales or measure chains. Recently, the theory of dynamic systems on time scales has gained impetus because it provides a framework which permits us to handle both continu-

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* Corresponding author.
E-mail address: pgwang@mail.hbu.edu.cn (P.G. Wang).
1 Present address: Department of Mathematics and Computer Science, Baoding Teachers College, China.

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ous and discrete dynamic systems simultaneously so that one can get some insight and a better understanding of the subtle differences of these two different systems [1].

Lakshmikantham and Leela [2] initiated the method of cone and cone-valued Lyapunov functions and developed the theory of differential inequalities. Since then, Akpan and Akinyele [3] discussed the ϕ_0-stability of comparison differential systems and gave some criteria of ϕ_0-stability of ordinary differential equations using method of cone-valued Lyapunov functions. Especially, they also gave a simple example which illustrated the advantage of using cone-valued Lyapunov functions. It successfully showed the stability of system whereas the method of scalar and vector Lyapunov functions failed. El-Sheikh and Soliman [4] discussed these notions of functional differential equations.

Recently, Lakshmikantham and Liu [5] gave the concept of hybrid systems, Wang and Liu [6,7] obtained the stability criteria for impulsive hybrid systems on time scales. The identifying characteristic of hybrid systems in general is that they incorporate both continuous components, usually called plants, which are governed by differential equations, and also digital components such as digital computers, sensors and actuators controlled by programs.

In this paper, we shall employ the method of cone-valued Lyapunov functions to investigate the ϕ_0-stability of impulsive hybrid systems on time scales, and give some stability results via comparison principle. At the same time, we give an example to illustrate our result.

2. Preliminaries

Let $\mathbb{T}$ be a time scale with $t_0 \geqslant 0$ as minimal element and no maximal element.

Definition 2.1. (See [1].) The mappings $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ defined as

$$\sigma(t) = \inf\{s \in \mathbb{T}:\ s > t\}$$

and

$$\rho(t) = \sup\{s \in \mathbb{T}:\ s < t\}$$

are called jump operators.

Definition 2.2. (See [1].) A nonmaximal element $t \in \mathbb{T}$ is said to be right-scattered (rs) if $\sigma(t) > t$ and right-dense (rd) if $\sigma(t) = t$. A nonminimal element $t \in \mathbb{T}$ is called left-scattered (ls) if $\rho(t) < t$ and left-dense (ld) if $\rho(t) = t$.

Definition 2.3. (See [1].) The graininess function $\mu : \mathbb{T} \to [0, \infty)$ is defined by

$$\mu\big(\sigma(t), t\big) = \sigma(t) - t.$$

For convenience, we denote it by $\mu^*(t)$. When $\mathbb{T} = \mathbb{Z}$, $\mu^*(t) \equiv 1$ and $\mathbb{T} = \mathbb{R}$, $\mu^*(t) \equiv 0$.

Definition 2.4. (See [1].) The mapping $g : \mathbb{T} \to X$ where X is a Banach space, is called rd continuous if

(i) it is continuous at each right-dense $t \in \mathbb{T}$,
(ii) at each left-dense point the left-sided limit $g(t^-)$ exists.

Let $C_{\mathrm{rd}}[\mathbb{T}, X]$ denote the set of rd-continuous mappings from $\mathbb{T}$ to X.