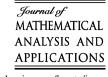




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Fixed points of generalized e-concave (generalized e-convex) operators and their applications [☆]

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Abstract

In this paper, we present the definitions of generalized e-concave operators and generalized e-convex operators, which are the generalizations of e-concave operators and e-convex operators, respectively. Without compactness or continuity assumption of generalized e-concave operators and generalized e-convex operators, we have proved the existence, uniqueness and monotone iterative techniques of their fixed points. Our results are even new to e-concave operators and e-convex operators. Finally, we apply the results to the singular boundary value problems for second order differential equations.

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Keywords: Generalized e-concave operator; Generalized e-convex operator; Fixed point; Two point boundary value problem; Increasing operator; Decreasing operator

1. Introduction

[1] states the definitions and properties of e-concave operators and e-convex operators which have been investigated by Guo and Lakshmikantham in [2]. More research can be found in [3]. In this paper, we present the definitions of generalized e-concave operators and generalized e-convex operators, which are the generalizations of e-concave operators and e-convex operators, respectively. Without compactness or continuity assumption of generalized e-concave operators

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and generalized e-convex operators, we have proved the existence, uniqueness and monotone iterative techniques of their fixed points. As corollaries, we also obtain the existence of fixed points of e-concave operators and e-convex operators. Finally, we apply the results to the singular boundary value problems for second order differential equations, which improved the previous results.

Let E be a real Banach space, P be a cone of E and " \leq " be the partial ordering defined by P, $e \in P - \{\theta\}$ and

 $C_e = \{x \in E \mid \text{there exist positive numbers } \alpha, \beta \text{ such that } \alpha e \leqslant x \leqslant \beta e\}.$

Set

$$E_e = \{x \in E \mid \text{there exists } \lambda > 0 \text{ such that } -\lambda e \leq x \leq \lambda e\},\$$

and

$$||x||_e = \inf\{\lambda > 0 \mid -\lambda e \le x \le \lambda e\}, \quad \forall x \in E_e.$$

It is easy to see that E_e becomes a normed linear space under the norm $\|.\|_e$. $\|.\|_e$ is called the e-norm of the element $x \in E_e$.

Recall that cone *P* is said to be normal if there exists a positive constant *N* such that $\theta \le x \le y$ implies $||x|| \le N||y||$, the smallest *N* is called the normal constant of *P*.

Definition 1.1. Let $A: P \to P$ be an operator and $e > \theta$. Suppose that

- (i) $Ae \in C_e$,
- (ii) there exists a real number $\eta = \eta(x, t) > 0$ such that

$$A(tx) \ge t(1+\eta)Ax, \quad \forall x \in C_e, \ 0 < t < 1. \tag{1.1}$$

Then A is called a generalized e-concave operator.

Similarly, if in the above definition, we replace (ii) by the following

(ii)'
$$A(tx) \le (t(1+\eta))^{-1}Ax$$
, $\forall x \in C_e$, $0 < t < 1$, (1.2)

then A is called a generalized e-convex operator.

Remark 1.1. (1.1) implies

$$A(\lambda x) \leqslant \lambda \left[1 + \eta \left(\lambda x, \frac{1}{\lambda} \right) \right]^{-1} Ax, \quad \forall x \in C_e, \ \lambda > 1;$$
 (1.3)

and (1.2) implies

$$A(\lambda x) \geqslant \lambda^{-1} \left[1 + \eta \left(\lambda x, \frac{1}{\lambda} \right) \right] Ax, \quad \forall x \in C_e, \ \lambda > 1.$$
 (1.4)

Conversely, (1.3) implies (1.1) and (1.4) implies (1.2).

Remark 1.2. If we replace the condition (i) in Definition 1.1 by the following

(i)' for any
$$x \in P - \{\theta\}$$
, $Ax \in C_{\theta}$,

then A is called an e-concave operator. Obviously, an e-concave operator is a generalized e-concave operator.

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