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Parametric duality on minimax programming involving generalized convexity in complex space [☆]

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Abstract

In this paper we employ generalized convexity of complex functions to establish several sufficient optimality conditions for minimax programming in complex spaces. Using such criteria, we constitute a parametrical dual, and establish the weak, strong, and strict converse duality theorems in the framework.

Keywords: Complex minimax programming; $(\mathcal{F},\rho,\theta)$-convex; $(\mathcal{F},\rho,\theta)$-pseudoconvex/quasiconvex; Duality

1. Introduction

Mathematical programming in complex space has many applications. In the electrical network, the alternating currents/voltages are using complex variable $z \in \mathbb{C}^n$ to stand for elements of network. The theory of complex programming is employed in variant fields of electric engineering, such as blind deconvolution, blind equalization, minimal entropy, optimal receiver, etc. (cf. Lai and Liu [14,15]).

Complex programming was firstly studied by Levinson [16] who considered linear programming in complex space. Shortly after Swarup and Sharma [22] studied linear fractional programming in complex space. Henceforth, many authors investigated nonlinear fractional or nonfractional complex programming in different viewpoints. See [1–15,17–19,21,24], etc. Recently, Chen, Lai and Schaible [5] have introduced a generalized Charnes–Cooper variable trans-

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formation (GCCT) to change fractional complex programming into a nonfractional programming problem, and have showed that the optimal solution of complex fractional programming problem can be reduced to an optimal solution of the equivalent nonfractional complex programming problem, and vice versa. While the existence of optimal solution, continuity, convexity and its various generalizations are valuable properties for the analysis and solution of such problems. However convexity and continuity are not sufficient, in general, to guarantee the existence of an optimal solution in programming problem. Usually one needs to assume compactness of feasible solutions. For example, we see that $f(x) = e^{-x}$ is convex and continuous in $\mathbb{R}$ but a minimum does not exist. While generalized convexity sometimes guarantees that a local minimum is a global minimum, and the existence of a minimum in case of noncompact feasible region is separate matter.

The purpose of this paper is to employ this idea of generalized convexity (see Chen and Lai [4], see also Lai and Liu [14,15]) to establish several sufficient optimality conditions in the framework of minimax programming problem in complex space. It is remarkable that a nonlinear complex function $f(z)$ cannot have a convex real part (cf. Ferrero [9, Proposition 3.1]), thus in our works, all nonlinear complex functions $f(z)$ in a programming problem are considered in the form $f(z, \bar{z})$ by variable z accompany with its conjugate complex $\bar{z}$. In this paper, we will constitute firstly a minimax programming in complex space in the following form:

$$(P_C) \quad \underset{\xi \in X}{\text{Minimize}} \quad \underset{\eta \in Y}{\text{Maximize}} \quad \operatorname{Re} \varphi(\xi, \eta)$$
$$\text{subject to} \quad -g(\xi) \in S \subset \mathbb{C}^p,$$

where Y is a compact subset of $\{\eta = (w, \bar{w}) \mid w \in \mathbb{C}^m\}$ in $\mathbb{C}^{2m}$, $\mathcal{S}$ is a polyhedral cone in $\mathbb{C}^p$, and for each $\eta \in Y$, the mappings

$$\varphi(\cdot, \eta) : \mathbb{C}^{2n} \to \mathbb{C} \quad \text{and} \quad g : \mathbb{C}^{2n} \to \mathbb{C}^p$$

are analytic over the manifold $X = \{\xi = (z, \bar{z}) \in \mathbb{C}^{2n} \mid z \in \mathbb{C}^n\}$.

Denote by $X_P = \{\xi = (z, \bar{z}) \in \mathbb{C}^{2n} \mid -g(\xi) \in \mathcal{S} \subset \mathbb{C}^p\}$ the feasible set of (P).

It is easy to verify that the manifold X is a closed convex cone over real field $\mathbb{R}$ (not over complex field $\mathbb{C}$). In order to have the convex real part of a nonlinear complex function, it needs to take function $f(\xi, \eta)$ with $\xi = (z, \bar{z}) \in \mathbb{C}^{2n}$ and $\eta = (w, \overline{w}) \in \mathbb{C}^{2m}$ in our requirement of complex minimax problem since any nonlinear analytic function $f(z)$, $z \in \mathbb{C}^n$, cannot have convex real part (cf. Ferreor [9], cf. also Lai and Liu [14,15]).

The special case of problem (P_C) is regarded as the real case. That is to consider a minimax programming problem in real variable in the form

$$(P_r) \quad \underset{x \in \mathbb{R}^n}{\text{Minimize}} \quad \underset{y \in Y}{\text{Maximize}} \quad f(x, y)$$
$$\text{subject to} \quad g(x) \leqslant 0,$$

where Y is a compact subset in $\mathbb{R}^m$, $f(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}^p$ are $\mathbb{C}^1$-mappings, that is, continuous differentiable functions.

Schmittedorf [20] established the necessary and sufficient optimality conditions for problem (P_r) under conditions of convexity. Later Weir [23] relaxed the convexity assumptions to be pseudoconvex/quasiconvex functions to establish sufficient conditions for optimal solution of (P_r), and then proved the weak, strong and strict converse duality theorems. Usually sufficient optimality conditions need extra assumptions in the converse of necessary optimality conditions. So several generalized convexities are constituted to establish the sufficient conditions for programming problem. Recently, Lai and Liu defined the generalized convexity, namely