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Asymptotic behavior of solutions for a class of predator–prey reaction–diffusion systems with time delays ☆

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Abstract

The aim of this paper is to investigate the asymptotic behavior of solutions for a class of three-species predator–prey reaction–diffusion systems with time delays under homogeneous Neumann boundary condition. Some simple and easily verifiable conditions are given to the rate constants of the reaction functions to ensure the convergence of the time-dependent solution to a constant steady-state solution. The conditions for the convergence are independent of diffusion coefficients and time delays, and the conclusions are directly applicable to the corresponding parabolic-ordinary differential system and to the corresponding system without time delays.

Keywords: Reaction–diffusion system; Predator–prey model; Time delay; Asymptotic behavior; Upper and lower solutions

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1. Introduction

Differential equations with time delays are traditionally formulated in the framework of ordinary differential systems. In recent years considerable attention has been given to parabolic systems with time delays, especially in relation to reaction–diffusion systems where the reaction functions depend on the unknown functions with time delays (see [1,2,4,5,8–24]). In this paper we investigate the asymptotic behavior of solutions for a class of three-species predator–prey reaction–diffusion systems with time delays in a bounded domain Ω in $\mathbf{R}^n$ under Neumann boundary condition. The system under the consideration is given in the form

$$\begin{cases} \partial u_1/\partial t = d_1 \Delta u_1 + u_1\big(a_1 - b_{11}u_1 - b_{12}u_2(x,t-\tau_2)\big), \\ \partial u_2/\partial t = d_2 \Delta u_2 + u_2\big(-a_2 + b_{21}u_1(x,t-\tau_1) - b_{22}u_2 - b_{23}u_3(x,t-\tau_3)\big), \\ \partial u_3/\partial t = d_3 \Delta u_3 + u_3\big(-a_3 + b_{32}u_2(x,t-\tau_2) - b_{33}u_3\big), \quad (x,t) \in \Omega \times (0,\infty), \\ \partial u_1/\partial \nu = \partial u_2/\partial \nu = \partial u_3/\partial \nu = 0, \quad (x,t) \in \partial\Omega \times (0,\infty), \\ u_i(x,t) = \eta_i(x,t) \geqslant 0, \quad (x,t) \in \Omega \times [-\tau_i, 0]\ (i=1,2,3), \end{cases} \tag{1.1}$$

where Δ is the Laplace operator, $\partial u_i/\partial \nu$ denotes the outward normal derivative of u_i on the boundary $\partial\Omega$ of Ω, the constants a_i, b_{ij}, d_i and τ_i satisfy $a_i > 0$, $b_{ij} > 0$, $d_i \geqslant 0$ and $\tau_i > 0$ for all $i, j = 1, 2, 3$, and the initial function $\eta_i(x,t)$ is Hölder continuous on $\overline{\Omega} \times [\tau_i, 0]$ $(i = 1, 2, 3)$.

Problem (1.1) arises in a predator–prey biological model for three species in which the third species is the predator of the second one and the second species is the predator of the first one. In biological terms, the unknown u_i represents the spatial density of the ith species at t in the habitat Ω and thus only nonnegative u_i is of interest. The coefficients d_1, d_2 and d_3 are the dispersal rates or diffusion coefficients, a_1 denotes the intrinsic growth rate of the first species, a_2 and a_3 denote the mortality rates of the second and the third species, respectively, b_{11}, b_{22} and b_{33} represent self-limitation rates, b_{12}, b_{21}, b_{23} and b_{32} represent interaction rates, and τ_i denotes the time delay (see [2,10]). The boundary conditions in (1.1) imply that the species are confined to Ω, i.e., there is no migration across the boundary of Ω.

By the method of upper and lower solutions we know that problem (1.1) has a unique global nonnegative solution (u_1, u_2, u_3). Moreover, if $\eta_i(x,0) \not\equiv 0$ then $u_i(x,t) > 0$ on $\overline{\Omega} \times (0,\infty)$ and if $\eta_i(x,0) \equiv 0$ then $u_i(x,t) \equiv 0$ on $\overline{\Omega} \times [0,\infty)$ (cf. [14]). The asymptotic behavior of the solution of (1.1) has been investigated in [2,10], and various sufficient conditions for the convergence of the solution to a constant steady-state solution are obtained. Clearly, a constant steady-state solution (c_1, c_2, c_3) of (1.1) is governed by

$$\begin{cases} c_1(a_1 - b_{11}c_1 - b_{12}c_2) = 0, \\ c_2(-a_2 + b_{21}c_1 - b_{22}c_2 - b_{23}c_3) = 0, \\ c_3(-a_3 + b_{32}c_2 - b_{33}c_3) = 0. \end{cases} \tag{1.2}$$

It is obvious that the above system possesses the trivial nonnegative solution $(0, 0, 0)$ and the semitrivial nonnegative solution $(a_1/b_{11}, 0, 0)$. If $a_1 b_{21} > a_2 b_{11}$, then it has also the semitrivial nonnegative solution $(\hat{c}_1, \hat{c}_2, 0)$ where

$$\hat{c}_1 = \frac{a_1 b_{22} + a_2 b_{12}}{b_{11} b_{22} + b_{12} b_{21}}, \qquad \hat{c}_2 = \frac{a_1 b_{21} - a_2 b_{11}}{b_{11} b_{22} + b_{12} b_{21}}, \tag{1.3}$$

and if

$$a_1 b_{21} b_{32} > a_2 b_{11} b_{32} + a_3 (b_{11} b_{22} + b_{12} b_{21}), \tag{1.4}$$