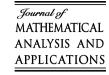




J. Math. Anal. Appl. 328 (2007) 295-301



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On some applications of the Briot–Bouquet differential subordination

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Received 3 March 2006
Available online 15 June 2006
Submitted by William F. Ames

Abstract

Recently Srivastava et al. [J. Dziok, H.M. Srivastava, Certain subclasses of analytic functions associated with the generalized hypergeometric function, Integral Transforms Spec. Funct. 14 (2003) 7–18; J. Dziok, H.M. Srivastava, Classes of analytic functions associated with the generalized hypergeometric function, Appl. Math. Comput. 103 (1999) 1–13; Y.C. Kim, H.M. Srivastava, Fractional integral and other linear operators associated with the Gaussian hypergeometric function, Complex Var. Theory Appl. 34 (1997) 293–312] introduced and studied a class of analytic functions associated with the generalized hypergeometric function. In the present paper, by using the Briot–Bouquet differential subordination, new results in this class are obtained.

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Keywords: Analytic functions; The generalized hypergeometric function; The Carlson–Shaffer operator; The Briot–Bouquet differential subordination

1. Introduction

Let \mathcal{A} denote the class of functions which are *analytic* in $\mathcal{U} = \mathcal{U}(1)$, where

$$\mathcal{U}(r) = \{ z \colon z \in \mathbb{C} \text{ and } |z| < r \}.$$

We denote by A_0 the class of functions $f \in A$ with the normalization f(0) = f'(0) - 1 = 0.

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We say that a function $f \in \mathcal{A}$ is *subordinate* to a function $F \in \mathcal{A}$ and write $f(z) \prec F(z)$, if and only if there exists a function $\omega \in \mathcal{A}$,

$$\omega(0) = 0, \qquad |\omega(z)| < 1 \quad (z \in \mathcal{U}),$$

such that

$$f(z) = F(\omega(z)) \quad (z \in \mathcal{U}).$$

Moreover, we say that f is subordinate to F in $\mathcal{U}(r)$, if $f(rz) \prec F(rz)$. We shall write

$$f(z) \prec_r F(z)$$

in this case. In particular, if F is univalent in \mathcal{U} , we have the following equivalence (cf. [10]):

$$f(z) \prec F(z) \iff f(0) = F(0) \text{ and } f(\mathcal{U}) \subset F(\mathcal{U}).$$

For analytic functions

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$
 and $g(z) = \sum_{n=0}^{\infty} b_n z^n$,

by f * g we denote the *Hadamard product or convolution* of f and g, defined by

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n.$$

Let $q, s \in \mathbb{N} = \{1, 2, ...\}$, $q \le s + 1$. For complex parameters $a_1, ..., a_q$ and $b_1, ..., b_s$ ($b_j \ne 0, -1, -2, ...; j = 1, ..., s$), we define the *generalized hypergeometric function* ${}_qF_s(a_1, ..., a_q; b_1, ..., b_s; z)$ by

$$_{q}F_{s}(a_{1},\ldots,a_{q};b_{1},\ldots,b_{s};z)=\sum_{n=0}^{\infty}\frac{(a_{1})_{n}\cdots(a_{q})_{n}}{(b_{1})_{n}\cdots(b_{s})_{n}}\frac{z^{n}}{n!}\quad(z\in\mathcal{U}),$$

where $(\lambda)_n$ is the Pochhammer symbol defined, in terms of the Gamma function Γ , by

$$(\lambda)_n = \frac{\Gamma(\lambda + n)}{\Gamma(\lambda)} = \begin{cases} 1 & (n = 0), \\ \lambda(\lambda + 1) \cdots (\lambda + n - 1) & (n \in \mathbb{N}). \end{cases}$$

Corresponding to a function $h(a_1, \ldots, a_a; b_1, \ldots, b_s; z)$ defined by

$$h(a_1, \ldots, a_a; b_1, \ldots, b_s; z) = z_a F_s(a_1, \ldots, a_a; b_1, \ldots, b_s; z),$$

we consider a linear operator

$$H(a_1,\ldots,a_q;b_1,\ldots,b_s):\mathcal{A}_0\to\mathcal{A}_0,$$

defined by the convolution:

$$H(a_1, \ldots, a_a; b_1, \ldots, b_s) f(z) = h(a_1, \ldots, a_a; b_1, \ldots, b_s; z) * f(z).$$

In particular, for s = 1 and q = 2 and $a_2 = 1$, we have the Carlson-Shaffer operator

$$\mathcal{L}(a_1, b_1) f(z) = H_1(a_1, 1; b_1) f(z),$$

which was introduced by Carlson and Shaffer [1] (see also [8]).

After some calculations we obtain

$$aH(a+1)f(z) = zH'(a)f(z) + (a-1)H(a)f(z),$$
(1)

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