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# Hilbert space-valued forward–backward stochastic differential equations with Poisson jumps and applications

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**Abstract**

In this paper, we study a class of Hilbert space-valued forward–backward stochastic differential equations (FBSDEs) with bounded random terminal times; more precisely, the FBSDEs are driven by a cylindrical Brownian motion on a separable Hilbert space and a Poisson random measure. In the case where the coefficients are continuous but not Lipschitz continuous, we prove the existence and uniqueness of adapted solutions to such FBSDEs under assumptions of weak monotonicity and linear growth on the coefficients. Existence is shown by applying a finite-dimensional approximation technique and the weak convergence theory. We also use these results to solve some special types of optimal stochastic control problems.
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## 1. Introduction

In finite-dimensional spaces, the existence and uniqueness of adapted solutions to forward–backward stochastic differential equations (FBSDEs for short) with Poisson jumps were established by SiTu [6], Yin and SiTu [9,10] via purely probabilistic approaches. They deal with the FBSDEs with Lipschitz continuous coefficients or with continuous coefficients in the case of fixed terminal time as well as in the case of random terminal time. Obviously, it is impossible to generalize these results to the infinite-dimensional spaces case for FBSDEs with non-Lipschitz continuous coefficients by using the smoothing technique (see, e.g., Yin and SiTu [10]). So we need to find another approach to solve such FBSDEs.

Hu and Peng [2] investigated a class of infinite-dimensional semi-linear backward stochastic evolution equations, and the so-called "mild solution" was given with the help of the Riesz representation theorem and an extended martingale representation theorem. Thereafter Hu and Peng [3] discussed semi-linear backward stochastic evolution equations and stochastic partial differential equations, and also proved the existence and uniqueness of adapted solutions by utilizing the extended martingale representation theorem and the stochastic Fubini theorem. These results were proved to be very useful in discussing stochastic Hamilton–Bellman–Jacobi equations (cf. [5]). Moreover, SiTu [8] considered a class of backward stochastic differential equations which have jumps and are driven by a $K$-valued Brownian motion and a Poisson random measure. The existence and uniqueness results were established, and some results were used to solve some optimal stochastic control problems with respect to certain BSDEs with jumps in Hilbert spaces.

In this paper, we are concerned with a class of FBSDEs with bounded random terminal times in an infinite-dimensional space driven by a cylindrical Brownian motion and a Poisson random measure. We give the existence and uniqueness results for such FBSDEs when the coefficients are continuous but not Lipschitz continuous, and some applications to optimal stochastic control problems. The proof of existence is based on the theory of weak convergence and the method of finite-dimensional approximation. It should be mentioned that Yor [11] showed the existence and uniqueness of strong solutions for finite horizon forward SDEs in a Hilbert space when the coefficient satisfies linear growth condition and Lipschitz condition. The meaning of the strong solution is actually identical with that of the adapted solution. Indeed, this result can be generalized to the case of forward SDEs with Poisson jumps. Furthermore, under some suitable conditions (for example, monotonicity condition and Lipschitz condition on the coefficients in [9]), we can use Itô's formula for $H$-valued cylindrical Brownian motion and Poisson random measure in [8] and the method of continuation given by Hu and Peng [4] to prove the existence and uniqueness theorem of solutions to $H$-valued FBSDEs with Poisson jumps. But as above mentioned, for those $H$-valued FBSDEs without Lipschitz continuous coefficients we cannot depend on the smoothing technique of [10] to solve them. Although some ideas of [1,8] are used throughout this work, the differences of framework and studied subject, require more assumptions and more arguments.

The paper is organized as follows: in Section 2, we give the preliminaries, including the definition of Hilbert space-valued cylindrical Brownian motion and the corresponding stochastic integral with respect to it; in Section 3, the existence and uniqueness of adapted solutions to Hilbert space-valued FBSDEs with Poisson jumps and with non-Lipschitz continuous coefficients are proved by partially adopting some ideas of Darling and Pardoux [1] and SiTu [8]. And a priori estimate and a uniqueness theorem of adapted solutions to above FBSDEs are also given. Finally, in Section 4 we use an example, which is a class of special FBSDEs without Lipschitz

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