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Robust variance-constrained H_∞ control for stochastic systems with multiplicative noises ☆

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Abstract

In this paper, the robust variance-constrained H_∞ control problem is considered for uncertain stochastic systems with multiplicative noises. The norm-bounded parametric uncertainties enter into both the system and output matrices. The purpose of the problem is to design a state feedback controller such that, for all admissible parameter uncertainties, (1) the closed-loop system is exponentially mean-square quadratically stable; (2) the individual steady-state variance satisfies given upper bound constraints; and (3) the prescribed noise attenuation level is guaranteed in an H_∞ sense with respect to the additive noise disturbances. A general framework is established to solve the addressed multiobjective problem by using a linear matrix inequality (LMI) approach, where the required stability, the H_∞ characterization and variance constraints are all easily enforced. Within such a framework, two additional optimization problems are formulated: one is to optimize the H_∞ performance, and the other is to minimize the weighted sum of the system state variances. A numerical example is provided to illustrate the effectiveness of the proposed design algorithm.

Keywords: Stability; H_∞ performance; Variance constraint; Stochastic system; Multiplicative noises; Linear matrix inequality

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1. Introduction

In many engineering control problems, the performance requirements are naturally expressed as the upper bounds on the steady-state variances [1,9,10,13,16]. The covariance control theory aims to solve the variance-constrained control problems while satisfying other performance indices, such as L_1, H_2, H_∞, pole placement, see, e.g., [4,8,11,16]. It has been shown that the covariance control approach is capable of solving multiobjective design problems, which has found applications in dealing with transient responses, round off errors in digital control, residence time/probability in aiming control problems, and stability robustness in the presence of parameter perturbations, see [16]. Such an advantage is based on the fact that several control design objectives, such as stability, time-domain and frequency-domain performance specifications, robustness and pole location, can be directly related to steady-state covariance of the closed-loop systems. Therefore, covariance control theory serves as a practical method for multiobjective control design as well as a foundation for linear system theory.

On the other hand, the control and filtering problems for stochastic systems with multiplicative noises (also called bilinear systems or systems with state-dependent noises) have recently received much attention, since many plants may be modelled by systems with multiplicative noises, and some characteristics of nonlinear systems can be closely approximated by models with multiplicative noises rather than by linearized models. So far, there have been several approaches to dealing with the control and filtering problems for stochastic systems with multiplicative noises, such as the linear matrix inequality (LMI) approach [18], the Riccati equation approach [20,22], to name just a few. It is worth emphasizing that, the covariance control problem has been initially studied for stochastic systems with multiplicative noises in [3,23]. In [3], Chung and Chang developed the coordinate transformation method to assign the state covariance for stochastic continuous-time systems with multiplicative noises. In [23], Yasuda et al. considered covariance control problem for stochastic continuous-time systems with multiplicative noises, where the solvability of an assignability condition and the robustness of covariance controllers were also discussed. However, when there exist modelling uncertainties and external disturbances, the issues of robust control and H_∞ disturbance rejection attenuation will need to be addressed, in addition to the expected steady-state variance constraints. Unfortunately, up to now, the robust H_∞ control problems with variance constraints have not yet been investigated for stochastic systems with multiplicative noises, and remains open and challenging, though similar problem has been studied in [19] for *linear* system.

It is our objective in this paper to propose an LMI approach to solving the robust variance-constrained H_∞ control problem for stochastic systems with both multiplicative noises and norm-bounded parameter uncertainties. We aim to design a state feedback controller such that, for all admissible parameter uncertainties, the closed-loop system is exponentially mean-square quadratically stable, the individual steady-state variance satisfies given upper bound constraints, and the prescribed noise attenuation level is guaranteed in an H_∞ sense with respect to the additive noise disturbances. We will show that all the three requirements can be ideally enforced within a unified LMI framework. In order to demonstrate the flexibility of the proposed framework, we will examine two types of the optimization problems that optimize either the H_∞ performance or the system state variances, and a numerical example is provided to illustrate the effectiveness of the proposed design algorithm.

It is worth pointing out that the work in this paper represents the first attempt to consider multiple performance objectives for stochastic systems with multiplicative noises. These objectives include individual variance constraints, performance robustness and H_∞ disturbance rejection