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Existence and Mann iterative approximations of nonoscillatory solutions of *n*th-order neutral delay differential equations

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Abstract

In this paper we consider the following nth-order neutral delay differential equation:

$$\frac{d^n}{dt^n}\big[x(t)+cx(t-\tau)\big]+(-1)^{n+1}f\big(t,x(t-\sigma_1),x(t-\sigma_2),\dots,x(t-\sigma_k)\big)=g(t),\quad t\geqslant t_0,$$

where n is a positive integer, $c\in\mathbb{R}$, $\tau>0$, $\sigma_i>0$ for $i=1,\dots,k$, $f\in C([t_0,\infty)\times\mathbb{R}^k,\mathbb{R})$ and $g\in C([t_0,\infty),\mathbb{R}^+)$. By employing the contraction mapping principle, we obtain several existence results of nonoscillatory solutions for the above equation, construct a few Mann-type iterative approximation schemes for these nonoscillatory solutions and establish several error estimates between the approximate solutions and the nonoscillatory solutions. In addition, we obtain some sufficient conditions for the existence of infinitely many nonoscillatory solutions. These results presented in this paper extend, improve and unify many known results due to Cheng and Annie [J.F. Cheng, Z. Annie, Existence of nonoscillatory solution to second order linear neutral delay equation, J. Systems Sci. Math. Sci. 24 (2004) 389–397 (in Chinese)], Graef, Yang and Zhang [J.R. Graef, B. Yang, B.G. Zhang, Existence of nonoscillatory and oscillatory solutions of neutral

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differential equations with positive and negative coefficients, Math. Bohem. 124 (1999) 87–102], Kulenović and Hadžiomerspahić [M.R.S. Kulenović, S. Hadžiomerspahić, Existence of nonoscillatory solution of second order linear neutral delay equation, J. Math. Anal. Appl. 228 (1998) 436–448; M.R.S. Kulenović, S. Hadžiomerspahić, Existence of nonoscillatory solution for linear neutral delay equation, Fasc. Math. 32 (2001) 61–72], Zhang and Yu [B.G. Zhang, J.S. Yu, On the existence of asymptotically decaying positive solutions of second order neutral differential equations, J. Math. Anal. Appl. 166 (1992) 1–11], Zhang [B.G. Zhang, On the positive solutions of a kind of neutral equations, Acta Math. Appl. Sinica 19 (1996) 222–230] and Zhou and Zhang [Y. Zhou, B.G. Zhang, Existence of nonoscillatory solutions of higher-order neutral differential equations with positive and negative coefficients, Appl. Math. Lett. 15 (2002) 867–874] and others. Some nontrivial examples are given to illustrate the advantages of our results.

Keywords: nth-order neutral delay differential equation; Nonoscillatory solution; Infinitely many nonoscillatory solutions; Contraction mapping; Mann iterative sequence; Error estimate

1. Introduction and preliminaries

In this paper, we are concerned with the nth-order neutral delay differential equation:

$$\frac{d^n}{dt^n}\big[x(t)+cx(t-\tau)\big]+(-1)^{n+1}f\big(t,x(t-\sigma_1),x(t-\sigma_2),\dots,x(t-\sigma_k)\big)=g(t),\quad t\geqslant t_0, \tag{1.1}$$

where n is a positive integer, $c\in\mathbb{R}$, $\tau>0$, $\sigma_i>0$ for $i=1,\dots,k$, $f\in C([t_0,\infty)\times\mathbb{R}^k,\mathbb{R})$ and $g\in C([t_0,\infty),\mathbb{R}^+)$ satisfy the following assumptions:

(H) there exist constants $M>N>0$ and functions $p,q\in C([t_0,\infty),\mathbb{R}^+)$ satisfying

$$\big|f(t,u_1,\dots,u_k)-f(t,\bar u_1,\dots,\bar u_k)\big| \leqslant p(t)\max\big\{|u_i-\bar u_i|:\ 1\leqslant i\leqslant k\big\},\quad t\in[t_0,\infty),\ u_i,\bar u_i\in[N,M],\ 1\leqslant i\leqslant k,$$

and

$$\big|f(t,u_1,\dots,u_k)\big|\leqslant q(t),\quad t\in[t_0,\infty),\ u_i\in[N,M],\ 1\leqslant i\leqslant k.$$

It is well known that the nonoscillatory and oscillatory solutions for various kinds of neutral delay differential equations are of both theoretical and practical interest [1–17]. In 1998 and 2001, Kulenović and Hadžiomerspahić [8,9] studied the first- and second-order neutral delay differential equations with positive and negative coefficients:

$$\frac{d}{dt}\big[x(t)+cx(t-\tau)\big]+Q_1(t)x(t-\sigma_1)-Q_2(t)x(t-\sigma_2)=0,\quad t\geqslant t_0, \tag{1.2}$$

and

$$\frac{d^2}{dt^2}\big[x(t)+cx(t-\tau)\big]+Q_1(t)x(t-\sigma_1)-Q_2(t)x(t-\sigma_2)=0,\quad t\geqslant t_0. \tag{1.3}$$

Under $c\neq\pm1$, $aQ_1(t)\geqslant Q_2(t)$ and other conditions, they obtained a sufficient condition for the existence of a nonoscillatory solution of Eq. (1.3). In 2002, Zhou and Zhang [17] extended the result in [8] to nth-order neutral functional differential equation with positive and negative coefficients: