Available online at www.sciencedirect.com
ScienceDirect
J. Math. Anal. Appl. 326 (2007) 1199–1211
Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS
www.elsevier.com/locate/jmaa

Extended Cesàro operators between Bloch-type spaces in the unit ball of $\mathbf{C}^n$ ☆

Xiaomin Tang

Department of Mathematics, Huzhou Teachers College, Huzhou, Zhejiang 313000, PR China

Received 13 February 2006
Available online 27 April 2006
Submitted by J.A. Ball

Abstract

In this paper, we characterize those holomorphic symbols φ in the unit ball of $\mathbf{C}^n$ for which the induced extended Cesàro operator $T_\varphi : \mathcal{B}_\omega \to \mathcal{B}_\mu$ (respectively, $\mathcal{B}_{\omega,0} \to \mathcal{B}_{\mu,0}$) is bounded or compact, where ω and μ are normal functions on $[0, 1)$. In addition, we obtain some properties of those spaces $\mathcal{B}_\omega$ and $\mathcal{B}_{\omega,0}$.

Keywords: Bloch-type space; Extended Cesàro operator; Unit ball

1. Introduction

Let $\mathbf{B} = \{z \in \mathbf{C}^n;\ |z| < 1\}$ be the unit ball of $\mathbf{C}^n$, and let $\partial\mathbf{B} = \{z \in \mathbf{C}^n;\ |z| = 1\}$ be its boundary. $H(\mathbf{B})$ denotes the family of all holomorphic functions on $\mathbf{B}$.

A positive continuous function ω on $[0, 1)$ is called normal if there are three constants $0 \leqslant \delta < 1$ and $0 < a < b < \infty$ such that

$$\frac{\omega(r)}{(1-r)^a} \text{ is decreasing} \quad \text{and} \quad \frac{\omega(r)}{(1-r)^b} \text{ is increasing} \quad \text{on } [\delta, 1). \tag{1.1}$$

☆ Supported by the National Natural Science Foundation of China (No. 10471039), the Natural Science Foundation of Zhejiang Province (No. M103104) and the Natural Science Foundation of Huzhou City (No. 2005YZ02).
E-mail address: txm@hutc.zj.cn.

doi:10.1016/j.jmaa.2006.03.082

Given ω we will extend it to $\mathbf{B}$ by $\omega(z)=\omega(|z|)$. A function $f\in H(\mathbf{B})$ is said to belong to the Bloch-type space $\mathcal{B}_\omega$ if

$$\|f\|_{\mathcal{B},\omega}=\sup_{z\in\mathbf{B}}\omega(z)\left|\nabla f(z)\right|<\infty;$$

and it is said to belong to the little Bloch-type space $\mathcal{B}_{\omega,0}$ if

$$\lim_{|z|\to 1}\omega(z)\left|\nabla f(z)\right|=0.$$

Here $\nabla f(z)=(\frac{\partial f}{\partial z_1},\dots,\frac{\partial f}{\partial z_n})$ is the complex gradient of f. It is easy to check that both $\mathcal{B}_\omega$ and $\mathcal{B}_{\omega,0}$ are Banach spaces under the norm

$$\|f\|_\omega=\left|f(0)\right|+\|f\|_{\mathcal{B},\omega},$$

and that $\mathcal{B}_{\omega,0}$ is a closed subspace of $\mathcal{B}_\omega$. When $\omega(r)=1-r^2$ and $\omega(r)=(1-r^2)^{1-\alpha}$ with $\alpha\in(0,1)$, two typical normal weights, the induced spaces $\mathcal{B}_\omega$ are the Bloch space and Lipschitz type space, respectively. And also, the space $\mathcal{B}_{(1-r^2)\log 1/(1-r^2)}$ is the weighted Bloch space.

Let $\mathbf{D}$ denote the open unit disc in the complex plane $\mathbf{C}$. For a holomorphic function $f(z)$ on $\mathbf{D}$ with Taylor expansion $f(z)=\sum_{j=0}^{\infty}a_jz^j$, the Cesàro operator acting on f is

$$C[f](z)=\sum_{j=0}^{+\infty}\left(\frac{1}{j+1}\sum_{k=0}^{j}a_k\right)z^j.$$

It is well know that $C[\cdot]$ acts as a bounded linear operator on various spaces of holomorphic functions, see [1–6], including the Hardy and Bergman spaces. But it is not bounded on the Bloch space (see [5,6]).

A little calculation shows $C[f](z)=\frac{1}{z}\int_0^z f(t)(\log\frac{1}{1-t})'\,dt$. Hence, on most holomorphic function spaces, $C[\cdot]$ is bounded if and only if the integral operator $f\mapsto\int_0^z f(t)(\log\frac{1}{1-t})'\,dt$ is bounded. From this point of view it is natural to consider the extended Cesàro operator T_φ with holomorphic symbol φ,

$$T_\varphi f(z)=\int_0^z f(t)\varphi'(t)\,dt. \tag{1.2}$$

The boundedness and compactness of this operator on Hardy spaces, Bergman spaces, Bloch-type spaces and Lipschitz spaces have been studied in [7–9].

For $f\in H(\mathbf{B})$, the radial derivative of f is $\Re f(z)=\sum_{j=1}^{n}z_j\frac{\partial f(z)}{\partial z_j}$. Given $\varphi\in H(\mathbf{B})$, the operator T_φ is defined by

$$T_\varphi f(z)=\int_0^1 f(tz)\Re\varphi(tz)\frac{dt}{t},\qquad f\in H(\mathbf{B}),\ z\in\mathbf{B}. \tag{1.3}$$

It is trivial that, when $n=1$, (1.3) is just (1.2). In the unit ball, Hu [10] got the characterization on φ for which the induced extended Cesàro operator is bounded or compact on $\mathcal{B}_{1-r^2}$ and $\mathcal{B}_{1-r^2,0}$, Xiao [11] obtained the sufficient and necessary conditions on φ such that T_φ is bounded or compact on $\mathcal{B}_{(1-r^2)^\alpha}$ and $\mathcal{B}_{(1-r^2)^\alpha,0}$, where $\alpha\in(0,\infty)$ but $\alpha\neq 1$. Stević [12] considered the boundedness of T_φ on $\mathcal{B}_{(1-r^2)^\alpha}$ with $\alpha\in(0,\infty)$, and Zhang [13] studied the same problems between $\mathcal{B}_{(1-r^2)^p}$ and $\mathcal{B}_{(1-r^2)^q}$ for $0<p,q<\infty$. And also, Hu discussed the boundedness and