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Using infinite matrices to approximate functions of class Lip(α, p) using trigonometric polynomials

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In memory of Professor Brian Kuttner, 1908–1992

Abstract

Given a function f in the class Lip(α, p) $(0 < \alpha \leqslant 1,\ p \geqslant 1)$, Chandra [P. Chandra, Trigonometric approximation of functions in L_p-norm, J. Math. Anal. Appl. 275 (2002) 13–26] approximated such an f by using trigonometric polynomials, which are the nth terms of either certain weighted mean or Nörlund mean transforms of the Fourier series representation for f. He showed that the degree of its approximation is $O(n^{-\alpha})$. In this paper we obtain the same degree of approximation for a more general class of lower triangular matrices, and deduce some of the results of [P. Chandra, Trigonometric approximation of functions in L_p-norm, J. Math. Anal. Appl. 275 (2002) 13–26] as corollaries.

Keywords: Class Lip(α, p); Trigonometric approximation; L_p-Norm

Let $\sigma_n(f)$ denote the nth term of the $(C, 1)$ transform of the partial sums of the Fourier series of a 2π-periodic function f. In 1937 Quade [4] proved that, if $f \in \text{Lip}(\alpha, p)$ for $0 < \alpha \leqslant 1$, then $\|f - \sigma_n(f)\|_p = O(n^{-\alpha})$ for either $p > 1$ and $0 < \alpha \leqslant 1$ or $p = 1$ and $0 < \alpha < 1$. He also showed that, if $p = \alpha = 1$, then $\|f - \sigma_n(f)\|_1 = O(n^{-1}\log(n+1))$. In a recent paper Chandra [2] extended the work of Quade and proved the following theorems, where $N_n(f)$ and

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$R_n(f)$ denote the nth terms of the Nörlund and weighted mean transforms of the sequences of partial sums, respectively.

Theorem 1. [2] *Let $f \in \operatorname{Lip}(\alpha, p)$ and let $\{p_n\}$ be a positive sequence such that*

$$(n+1)p_n = O(P_n). \tag{1}$$

If either

(i) $p > 1$, $0 < \alpha \leqslant 1$, *and*
(ii) $\{p_n\}$ *is monotonic, or*

(i) $p = 1$, $0 < \alpha < 1$, *and*
(ii) $\{p_n\}$ *is a nondecreasing sequence,*

then

$$\|f - N_n(f)\|_p = O(n^{-\alpha}). \tag{2}$$

Theorem 2. [2] *Let $f \in \operatorname{Lip}(\alpha, p)$ and let $\{p_n\}$ be positive. Suppose that either*

(i) $p > 1$, $0 < \alpha \leqslant 1$, *and*
(ii) $$\sum_{k=0}^{n-1} \left| \Delta\left(\frac{P_k}{k+1}\right)\right| = O\left(\frac{P_n}{n+1}\right), \quad \textit{or}$$

(i) $p = 1$, $0 < \alpha < 1$ *and*
(ii) $\{p_n\}$ *with* (1) *is positive and nondecreasing. Then*

$$\|f - R_n(f)\|_p = O(n^{-\alpha}). \tag{3}$$

Theorem 3. [2] *Let $f \in \operatorname{Lip}(1, 1)$ and let $\{p_n\}$ be positive, satisfy* (1), *and be such that*

$$(n+1)^{-\eta} p_n \quad \textit{is nondecreasing for some } \eta > 0. \tag{4}$$

Then

$$\|f - R_n(f)\|_1 = O(n^{-1}). \tag{5}$$

In this paper we extend some of the results of Chandra to more general classes of triangular matrix methods.

For a given $f \in L_p := L_p[0, 2\pi]$, $p \geqslant 1$, let

$$s_n(f) := s_n(f; x) = \frac{1}{2}a_0 + \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx) := \sum_{k=0}^{n} u_k(f; x).$$

The integral modulus of continuity of f is defined by

$$\omega_p(\delta; f) := \sup_{0<|h|\leqslant\delta} \left\{ \frac{1}{2\pi} \int_0^{2\pi} \left| f(x+h) - f(x)\right|^p dx \right\}^{1/p}.$$