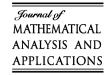


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# A general iterative method for nonexpansive mappings in Hilbert spaces

Giuseppe Marino a, Hong-Kun Xu b,\*,1

 a Dipartimento di Matematica, Universita della Calabria, 87036 Arcavacata di Rende (Cs), Italy
 b School of Mathematical Sciences, University of KwaZulu-Natal, Westville Campus, Private Bag X54001, Durban 4000, South Africa

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#### Abstract

Let H be a real Hilbert space. Consider on H a nonexpansive mapping T with a fixed point, a contraction f with coefficient  $0 < \alpha < 1$ , and a strongly positive linear bounded operator A with coefficient  $\bar{\gamma} > 0$ . Let  $0 < \gamma < \bar{\gamma}/\alpha$ . It is proved that the sequence  $\{x_n\}$  generated by the iterative method  $x_{n+1} = (I - \alpha_n A)Tx_n + \alpha_n \gamma f(x_n)$  converges strongly to a fixed point  $\tilde{x} \in \text{Fix}(T)$  which solves the variational inequality  $\langle (\gamma f - A)\tilde{x}, x - \tilde{x} \rangle \leq 0$  for  $x \in \text{Fix}(T)$ . © 2005 Elsevier Inc. All rights reserved.

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#### 1. Introduction

Iterative methods for nonexpansive mappings have recently been applied to solve convex minimization problems; see, e.g., [1,4,5,7,8] and the references therein. A typical

<sup>\*</sup> Corresponding author.

E-mail addresses: gmarino@unical.it (G. Marino), xuhk@ukzn.ac.za (H.-K. Xu).

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problem is to minimize a quadratic function over the set of the fixed points of a nonexpansive mapping on a real Hilbert space H:

$$\min_{x \in C} \frac{1}{2} \langle Ax, x \rangle - \langle x, b \rangle, \tag{1}$$

where C is the fixed point set of a nonexpansive mapping T on H and b is a given point in H. Assume A is strongly positive; that is, there is a constant  $\bar{\gamma} > 0$  with the property

$$\langle Ax, x \rangle \geqslant \bar{\gamma} \|x\|^2 \quad \text{for all } x \in H.$$
 (2)

Recall that  $T: H \to H$  is nonexpansive if  $||Tx - Ty|| \le ||x - y||$  for all  $x, y \in H$ . The set of fixed points of T is the set  $Fix(T) := \{x \in H : Tx = x\}$ . We assume that  $Fix(T) \ne \emptyset$  and C = Fix(T). It is well known that Fix(T) is closed convex (cf. [2]). In [5] (see also [7]), it is proved that the sequence  $\{x_n\}$  defined by the iterative method below, with the initial guess  $x_0 \in H$  chosen arbitrarily,

$$x_{n+1} = (I - \alpha_n A) T x_n + \alpha_n b, \quad n \geqslant 0, \tag{3}$$

converges strongly to the unique solution of the minimization problem (1) provided the sequence  $\{\alpha_n\}$  satisfies certain conditions that will be made precise in Section 3.

On the other hand, Moudafi [3] introduced the viscosity approximation method for non-expansive mappings (see [6] for further developments in both Hilbert and Banach spaces). Let f be a contraction on H. Starting with an arbitrary initial  $x_0 \in H$ , define a sequence  $\{x_n\}$  recursively by

$$x_{n+1} = (1 - \sigma_n)Tx_n + \sigma_n f(x_n), \quad n \geqslant 0,$$
(4)

where  $\{\sigma_n\}$  is a sequence in (0, 1). It is proved [3,6] that under certain appropriate conditions imposed on  $\{\sigma_n\}$ , the sequence  $\{x_n\}$  generated by (4) strongly converges to the unique solution  $x^*$  in C of the variational inequality

$$\langle (I-f)x^*, x-x^* \rangle \geqslant 0, \quad x \in C.$$
 (5)

In this paper we will combine the iterative method (3) with the viscosity approximation method (4) and consider the following general iterative method:

$$x_{n+1} = (I - \alpha_n A) T x_n + \alpha_n \gamma f(x_n), \quad n \geqslant 0.$$
 (6)

We will prove in Section 3 that if the sequence  $\{\alpha_n\}$  of parameters satisfies appropriate conditions, then the sequence  $\{x_n\}$  generated by (6) converges strongly to the unique solution of the variational inequality

$$\langle (A - \gamma f)x^*, x - x^* \rangle \geqslant 0, \quad x \in C,$$
 (7)

which is the optimality condition for the minimization problem

$$\min_{x \in C} \frac{1}{2} \langle Ax, x \rangle - h(x),$$

where *h* is a potential function for  $\gamma f$  (i.e.,  $h'(x) = \gamma f(x)$  for  $x \in H$ ).

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