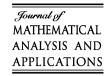


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# Higher-order spectral analysis and weak asymptotic stability of convex processes

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#### Abstract

This paper deals with the asymptotic stability analysis of a discrete dynamical inclusion whose right-hand side is a convex process. We provide necessary and sufficient conditions for weak asymptotic stability, and obtain sharp estimates for the asymptotic null-controllability set. These estimates involve not only standard, but also higher-order spectral information on the convex process and its adjoint.

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#### 1. Introduction

This paper deals with the asymptotic stability analysis of a discrete dynamical system of the form

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$$x(k+1) \in F(x(k)), \quad \forall k = 0, 1, \dots$$

As state space, consider a real Hilbert space H with inner product  $\langle \cdot, \cdot \rangle$  and associated norm  $\| \cdot \|$ . The multivalued operator  $F : H \rightrightarrows H$  is assumed to be a convex process in the sense that

$$\operatorname{gr} F = \{(s, v) \in H \times H \colon v \in F(s)\}\$$

is a convex cone containing the origin. This geometric property imposed on the graph of F amounts to saying that

$$0 \in F(0)$$
,

$$F(\alpha s) = \alpha F(s), \quad \forall \alpha > 0, \ \forall s \in H,$$

$$F(s_1) + F(s_2) \subset F(s_1 + s_2), \quad \forall s_1, s_2 \in H.$$

A trajectory of F refers to a sequence  $x : \mathbb{N} \to H$  satisfying the evolution law (1). Thus,

$$S_F(\xi) = \{x : \mathbb{N} \to H : x \text{ solves } (1) \text{ and } x(0) = \xi \}$$

corresponds to the set of all trajectories of F emanating from the initial state  $\xi \in H$ . Observe that the multivalued operator  $S_F : H \rightrightarrows H^{\mathbb{N}}$  enjoys the same properties as F, namely, normalization, positive homogeneity, and super-additivity.

**Definition 1.1.** F is said to be weakly asymptotically stable if

$$\forall \xi \in H$$
,  $\exists x \in S_F(\xi)$  such that  $\lim_{k \to \infty} x(k) = 0$ ,

that is to say, from every initial state emanates a trajectory of F that, in the long run, becomes arbitrarily close to the origin.

Weak asymptotic stability is a concept that speaks by itself and does not need any further introduction. Definition 1.1 has been considered by authors like Phat [10,11] and Smirnov [12], among others. The purpose of this note is not only providing necessary and sufficient conditions for weak asymptotic stability, but also deriving sharp estimates for the set

$$\mathcal{K}_{\infty}(F) = \Big\{ \xi \in H \colon \lim_{k \to \infty} x(k) = 0 \text{ for some } x \in S_F(\xi) \Big\}.$$

We say that  $\mathcal{K}_{\infty}(F)$  is the *asymptotic null-controllability set* of F. We are borrowing the terminology of control theory because (1) can be seen as a generalization of the control model

$$x(k+1) = Ax(k) + Bu(k), \quad u(k) \in P$$

where P is a closed convex cone in a given Hilbert space, and A and B are continuous linear operators.

Two remarks are useful for putting our study in the right perspective: firstly,  $\mathcal{K}_{\infty}(F)$  is a convex cone containing the origin; and, secondly,

$$\mathcal{K}_{\infty}(F) \subset \operatorname{dom} S_F \subset \operatorname{dom} F$$
,

with dom  $F = \{ \xi \in H \colon F(\xi) \neq \emptyset \}$  and dom  $S_F = \{ \xi \in H \colon S_F(\xi) \neq \emptyset \}$  being the domains of F and  $S_F$ , respectively. Needless to say, the convex process F cannot be weakly asymptotically stable unless it is nonempty-valued everywhere.

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