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Higher-order spectral analysis and weak asymptotic stability of convex processes

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Abstract

This paper deals with the asymptotic stability analysis of a discrete dynamical inclusion whose right-hand side is a convex process. We provide necessary and sufficient conditions for weak asymptotic stability, and obtain sharp estimates for the asymptotic null-controllability set. These estimates involve not only standard, but also higher-order spectral information on the convex process and its adjoint.

Keywords: Convex process; Discrete dynamical inclusion; Eigenvalue analysis; Asymptotic stability

1. Introduction

This paper deals with the asymptotic stability analysis of a discrete dynamical system of the form

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$$x(k+1) \in F\big(x(k)\big), \quad \forall k = 0, 1, \ldots. \tag{1}$$

As state space, consider a real Hilbert space H with inner product $\langle \cdot, \cdot \rangle$ and associated norm $\| \cdot \|$. The multivalued operator $F : H \rightrightarrows H$ is assumed to be a convex process in the sense that

$$\operatorname{gr} F = \big\{(s, v) \in H \times H \colon v \in F(s)\big\}$$

is a convex cone containing the origin. This geometric property imposed on the graph of F amounts to saying that

$$0 \in F(0),$$
$$F(\alpha s) = \alpha F(s), \quad \forall \alpha > 0, \ \forall s \in H,$$
$$F(s_1) + F(s_2) \subset F(s_1 + s_2), \quad \forall s_1, s_2 \in H.$$

A trajectory of F refers to a sequence $x : \mathbb{N} \to H$ satisfying the evolution law (1). Thus,

$$S_F(\xi) = \big\{x : \mathbb{N} \to H \colon x \text{ solves (1) and } x(0) = \xi\big\}$$

corresponds to the set of all trajectories of F emanating from the initial state $\xi \in H$. Observe that the multivalued operator $S_F : H \rightrightarrows H^{\mathbb{N}}$ enjoys the same properties as F, namely, normalization, positive homogeneity, and super-additivity.

Definition 1.1. F is said to be weakly asymptotically stable if

$$\forall \xi \in H, \quad \exists x \in S_F(\xi) \quad \text{such that} \quad \lim_{k \to \infty} x(k) = 0,$$

that is to say, from every initial state emanates a trajectory of F that, in the long run, becomes arbitrarily close to the origin.

Weak asymptotic stability is a concept that speaks by itself and does not need any further introduction. Definition 1.1 has been considered by authors like Phat [10,11] and Smirnov [12], among others. The purpose of this note is not only providing necessary and sufficient conditions for weak asymptotic stability, but also deriving sharp estimates for the set

$$\mathcal{K}_\infty(F) = \Big\{\xi \in H \colon \lim_{k \to \infty} x(k) = 0 \text{ for some } x \in S_F(\xi)\Big\}.$$

We say that $\mathcal{K}_\infty(F)$ is the *asymptotic null-controllability set* of F. We are borrowing the terminology of control theory because (1) can be seen as a generalization of the control model

$$x(k+1) = Ax(k) + Bu(k), \quad u(k) \in P,$$

where P is a closed convex cone in a given Hilbert space, and A and B are continuous linear operators.

Two remarks are useful for putting our study in the right perspective: firstly, $\mathcal{K}_\infty(F)$ is a convex cone containing the origin; and, secondly,

$$\mathcal{K}_\infty(F) \subset \operatorname{dom} S_F \subset \operatorname{dom} F,$$

with $\operatorname{dom} F = \{\xi \in H \colon F(\xi) \neq \emptyset\}$ and $\operatorname{dom} S_F = \{\xi \in H \colon S_F(\xi) \neq \emptyset\}$ being the domains of F and S_F, respectively. Needless to say, the convex process F cannot be weakly asymptotically stable unless it is nonempty-valued everywhere.