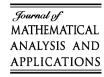


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# On semilinear elliptic equations involving concave—convex nonlinearities and sign-changing weight function

Tsung-Fang Wu<sup>1</sup>

Center for General Education, Southern Taiwan University of Technology, Tainan 71005, Taiwan
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#### Abstract

In this paper, we study the combined effect of concave and convex nonlinearities on the number of positive solutions for semilinear elliptic equations with a sign-changing weight function. With the help of the Nehari manifold, we prove that there are at least two positive solutions for Eq.  $(E_{\lambda,f})$  in bounded domains.

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#### 1. Introduction

In this paper, we consider the multiplicity results of positive solutions of the following semilinear elliptic equation:

$$\begin{cases}
-\Delta u = u^p + \lambda f(x)u^q & \text{in } \Omega, \\
0 \leqslant u \in H_0^1(\Omega),
\end{cases} (E_{\lambda,f})$$

E-mail address: tfwu@mail.stut.edu.tw.

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where  $\Omega$  is a bounded domain in  $\mathbb{R}^N$ ,  $0 < q < 1 < p < 2^*$   $(2^* = \frac{N+2}{N-2})$  if  $N \ge 3$ ,  $2^* = \infty$  if N = 2),  $\lambda > 0$  and  $f : \overline{\Omega} \to \mathbb{R}$  is a continuous function which change sign in  $\overline{\Omega}$ . Associated with Eq.  $(E_{\lambda, f})$ , we consider the energy functional  $J_{\lambda}$ , for each  $u \in H_0^1(\Omega)$ ,

$$J_{\lambda}(u) = \frac{1}{2} \int_{\Omega} |\nabla u|^2 dx - \frac{1}{p+1} \int_{\Omega} |u|^{p+1} dx - \frac{\lambda}{q+1} \int_{\Omega} f(x) |u|^{q+1} dx.$$

It is well known that the solutions of Eq.  $(E_{\lambda,f})$  are the critical points of the energy functional  $J_{\lambda}$  (see Rabinowitz [12]).

The fact that the number of positive solutions of Eq.  $(E_{\lambda,f})$  is affected by the concave and convex nonlinearities has been the focus of a great deal of research in recent years. If the weight function  $f(x) \equiv 1$ , the authors Ambrosetti et al. [2] have investigated Eq.  $(E_{\lambda,1})$ . They found that there exists  $\lambda_0 > 0$  such that Eq.  $(E_{\lambda,1})$  admits at least two positive solution for  $\lambda \in (0, \lambda_0)$ , has a positive solution for  $\lambda = \lambda_0$  and no positive solution exists for  $\lambda > \lambda_0$ . Actually, Adimurthy et al. [1], Damascelli et al. [7], Ouyang and Shi [11], and Tang [16] proved that there exists  $\lambda_0 > 0$  such that Eq.  $(E_{\lambda,1})$  in the unit ball  $B^N(0;1)$  has exactly two positive solution for  $\lambda \in (0,\lambda_0)$ , has exactly one positive solution for  $\lambda = \lambda_0$  and no positive solution exists for  $\lambda > \lambda_0$ .

The purpose of this paper is to consider the multiplicity of positive solution of Eq.  $(E_{\lambda,f})$  for a changing sign potential function f(x). We prove that Eq.  $(E_{\lambda,f})$  has at least two positive solutions for  $\lambda$  is sufficiently small.

**Theorem 1.** There exists  $\lambda_0 > 0$  such that for  $\lambda \in (0, \lambda_0)$ , Eq.  $(E_{\lambda, f})$  has at least two positive solutions.

Among the other interesting problems which are similar of Eq.  $(E_{\lambda,f})$  for q=0, Bahri [3], Bahri and Berestycki [4], and Struwe [13] have investigated the following equation:

$$\begin{cases}
-\Delta u = |u|^{p-1}u + f(x) & \text{in } \Omega, \\
u \in H_0^1(\Omega),
\end{cases}$$
(E<sub>f</sub>)

where  $f \in L^2(\Omega)$  and  $\Omega$  is a bounded domain in  $\mathbb{R}^N$ . They found that Eq.  $(E_f)$  possesses infinitely many solutions. Furthermore, Cîrstea and Rădulescu [5], Cao and Zhou [6], and Ghergu and Rădulescu [10] have been investigated the analogue Eq.  $(E_f)$  in  $\mathbb{R}^N$ .

This paper is organized as follows. In Section 2, we give some notations and preliminaries. In Section 3, we prove that Eq.  $(E_{\lambda,f})$  has at least two positive solutions for  $\lambda$  sufficiently small.

#### 2. Notations and preliminaries

Throughout this section, we denote by *S* the best Sobolev constant for the embedding of  $H_0^1(\Omega)$  in  $L^{p+1}(\Omega)$ . Now, we consider the Nehari minimization problem: for  $\lambda > 0$ ,

$$\alpha_{\lambda}(\Omega) = \inf \{ J_{\lambda}(u) \mid u \in \mathbf{M}_{\lambda}(\Omega) \},$$

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