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Mean ergodicity of positive operators in *KB*-spaces

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Abstract

We prove that any positive power bounded operator T in a *KB*-space E which satisfies

$$\lim_{n\to\infty} \operatorname{dist}\left(\frac{1}{n}\sum_{k=0}^{n-1} T^k x, [-g, g] + \eta B_E\right) = 0 \quad \big(\forall x \in E,\ \|x\| \leqslant 1\big), \tag{1}$$

where B_E is the unit ball of E, $g \in E_+$, and $0 \leqslant \eta < 1$, is mean ergodic and its fixed space $\mathrm{Fix}(T)$ is finite dimensional. This generalizes the main result of [E.Yu. Emelyanov, M.P.H. Wolff, Mean lower bounds for Markov operators, Ann. Polon. Math. 83 (2004) 11–19]. Moreover, under the assumption that E is a σ-Dedekind complete Banach lattice, we prove that if, for any positive power bounded operator T, the condition (1) implies that T is mean ergodic then E is a *KB*-space.

Keywords: *KB*-space; Positive operator; Mean ergodic operator

1. Introduction

It was proved recently [4] that if T is a Markov operator on an L^1-space then T is mean ergodic and satisfies $\dim \mathrm{Fix}(T) < \infty$ whenever there exist a function $h \in L^1_+$ and a real $0 \leqslant \eta < 1$ such that

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$$\lim_{n\to\infty}\left\|\left(h-\frac{1}{n}\sum_{k=0}^{n-1}T^k f\right)_+\right\|\leqslant\eta$$

for every density f. In this paper, we extend this result to any positive power bounded operator on a *KB*-space. Moreover, we show that this property of positive operators characterizes *KB*-spaces among σ-Dedekind complete Banach lattices. The class of *KB*-spaces is large enough, it contains L^1-spaces as well as reflexive Banach lattices, for instance all L^p-spaces for $1 < p < \infty$. The principal tool in the proof of the main results of [4] was using the additivity of the norm on the positive part of the L_1-space. Since this is no longer the case for a general *KB*-space, we use different ideas in the present paper. Our notation and terminology are standard, we follow the books [1,8,10].

2. The main results

First we fix some necessary notion and definitions. Let E be a Banach lattice. For $x \leqslant y$ in E, we denote by $[x, y]$ the order interval $\{z \in E\colon\ x \leqslant z \leqslant y\}$, and by $B_E = \{z \in E\colon\ \|z\| \leqslant 1\}$ the closed unit ball of E. Given an element $x \in E$ and a nonempty subset $A \subseteq E$,

$$\operatorname{dist}(x, A) := \inf\bigl\{\|x-a\|\colon\ a \in A\bigr\}$$

denotes the distance between x and A. For any operator T on E, $\operatorname{Fix}(T)$ denotes the space of all fixed vectors of T, and $\mathcal{A}_n^T$ denotes the Cesàro means of T, i.e.,

$$\mathcal{A}_n^T = \frac{1}{n}\sum_{k=0}^{n-1}T^k.$$

An operator T on E is called *mean ergodic* if the sequence $(\mathcal{A}_n^T x)_n$ is norm convergent for all $x \in E$, and we call T *power bounded* if $\sup_{n\geqslant 0}\|T^n\| < \infty$.

If T is a positive operator on E, then $x \in E$ is called a *positive fixed vector of maximal support* if $x \in \operatorname{Fix}(T) \cap E_+$ and every $y \in \operatorname{Fix}(T) \cap E_+$ is contained in the band generated by x. An element $x \in E_+$ is called a *quasi-interior point* if the order ideal $E_x := \bigcup_{n=1}^{\infty}[-nx, nx]$ generated by x is norm-dense in E.

Theorem 1. *Let E be a KB-space, T be a positive power bounded operator in E, W be a weakly compact subset of E, and $\eta \in \mathbb{R}$, $0 \leqslant \eta < 1$ be such that*

$$\lim_{n\to\infty}\operatorname{dist}\bigl(\mathcal{A}_n^T x, W + \eta B_E\bigr) = 0$$

for any $x \in B_E$. Then T is mean ergodic.

Proof. The first part of this proof is motivated by the proof of Theorem 5.3 in Räbiger's paper [9].

Without lost of generality we may assume that E has a quasi-interior point. Indeed, for any $x \in E$, $x \neq 0$, we consider the closed order ideal F generated by $\{T^n|x|\colon\ n \geqslant 0\}$, instead of E. Then F is a *KB*-space [10, Proposition II.5.15] with a quasi-interior point $\sum_{n\geqslant 0}2^{-n}T^n|x|$ and $T(F) \subseteq F$. Moreover, F is a projection band in E [8, Corollary 2.2.4]. If $P : E \to F$ denotes the corresponding band projection, then

$$\lim_{n\to\infty}\operatorname{dist}\bigl(\mathcal{A}_n^T z, P(W) + \eta B_F\bigr) = 0 \quad (\forall z \in B_F).$$