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Invited Papers

Analytical method for gain in erbium doped fiber amplifier with pumb excited state absorption

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ABSTRACT

Analytical solutions to rate and propagation equations describing gain in terms of photon intensities, transition rates, and absorption and emission cross-sections is determined in radially symmetric and longitudinally uniform monomode erbium doped silica fiber in presence of pump excited-state absorption effect for a pumping wavelength of 514.5 nm under the steady-state conditions with the help of a homogenous four-band transition scheme.

1. Introduction

One of the major events in the history of optical telecommunications is the advent of Erbium-Doped Fiber Amplifier (EDFA). It provided new life to the optical fiber transmission that allow high bit-rate and dense wavelength division multiplexing over ultra long-haul terrestrial and submarine optical communication networks operating at the principal telecom window around 1550 nm. The success of the EDFA is attributed to the 100% radiative transition between metastable state and ground state of erbium ions in silica fiber with a relatively long fluorescence lifetime of ≈10 ms [1,2]. Ever since the first successful gain demonstration (20–30 dB) of EDFA in 1987 [3,4], gains as high as 54 dB have been achieved with corresponding low noise figure of 3.1 dB [5].

However, there are a multitude of complex effects that degrade the overall gain performance of EDFA. One of the dominant loss mechanisms having detrimental effect on the signal amplification is the occurrence of pump excited state absorption (ESA) which severely limits the gain performance of an EDFA pumped at certain wavelengths. For example, ESA is prominent at 514.5 nm pumping wavelength which limits the gain availability of the amplifier by depleting the metastable state and making inefficient use of pump energy [6].

During last two decades, a veritable explosion of theoretical models has taken place for a variety of applications of EDFA. However, a fundamental limitation of these models is that they give solutions in numerical form. On the other hand, analytical models derived from rate and propagation equations provide considerable physical insight into the amplifier characteristics such as the length dependence of amplifier gain and the spectral dependence of gain saturation. There have been strenuous efforts to analytically model the gain in EDFA [7–27], yet no complete analytical solution has been achieved that could describe the gain in a closed form. Among these analytical models, some important models have ignored ESA [12–14]; therefore, obtaining analytical solution to EDFA gain with ESA still remains a challenge.

The objective of this work is to present, an analytical solution for EDFA gain obtained directly by solving the rate and propagation equations in presence of ESA effect with the help of a homogeneous four-band transition scheme corresponding to the pumping wavelength regime of 514.5 nm. In Section 2, significant energy transitions involved in an erbium doped silicon fiber are given when effect of ESA is included in the energy band scheme. Solutions of rate and propagation equations are obtained in Section 3. In Section 4 focus is on analytical gain formalism. Section 4 summarizes the conclusions.

2. Significant energy transitions

The energy bands typically associated with Er^{3+}-ion dopant in a vitreous host corresponding to pumping configuration of 514.5 nm are $^4I_{15/2}$, $^4I_{13/2}$, $^2H_{11/2}$ and $^4G_{15/2}$ as schematically illustrated in Fig. 1. The following radiative and non-radiative absorption and emission transitions associated with the electrons in energy bands occur significantly [28,29]:

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(1) $^4I_{15/2} \leftrightarrow {}^2H_{11/2}$, the stimulated ground state absorption (GSA) and emission between ground state and pump state at the pump wavelength 514.5 nm.
(2) $^4I_{15/2} \leftrightarrow {}^4I_{13/2}$, the stimulated GSA and amplification between ground state and metastable state at the signal wavelength 1530 nm.
(3) $^4I_{13/2} \rightarrow {}^4G_{11/2}$, the pump photon ESA from the metastable state to the upper excited state.
(4) $^4I_{13/2} \rightarrow {}^4I_{15/2}$, the spontaneous radiative decay from the metastable state to the ground state.
(5) $^2H_{11/2} \rightarrow {}^4I_{13/2}$, the spontaneous nonradiative multiphonons decay from the pump state to the metastable state.
(6) $^4G_{11/2} \rightarrow {}^4I_{13/2}$, the spontaneous nonradiative multiphonon decay from the upper excited state to the pump state.
(7) $^2H_{11/2} \rightarrow {}^4I_{15/2}$, the spontaneous fast nonradiative decay from the pump state to the ground state.

3. Solution of rate and propagation equations

The rate equations describe the effects of absorption, stimulated emission, spontaneous emission, and multiphonon transitions on the population densities of Er^{3+} ions in silica host at individual levels. The interaction of the optical field with erbium ions is described by the multilevel transition rate equation system

$$\frac{dN_i}{dt} = \hat{R} \cdot \vec{N}_i. \tag{1}$$

Here the transition rate constant operator $\hat{R}$ expressed in matrix representation

$$\hat{R} = \begin{pmatrix} -u(\eta_p I_p \sigma_{13} + \eta_s I_s \sigma_{12}) & u\eta_s I_s \sigma_{21} + \frac{1}{\tau_{21}} & u\eta_p I_p \sigma_{31} + \frac{1}{\tau_{31}} & 0 \\ u\eta_s I_s \sigma_{12} & -\left(u\eta_s I_s \sigma_{21} + u\eta_p I_p \sigma_{24} + \frac{1}{\tau_{21}}\right) & \frac{1}{\tau_{32}} & \frac{1}{\tau_{42}} \\ u\eta_p I_p \sigma_{13} & 0 & -\left(u\eta_p I_p \sigma_{31} + \frac{1}{\tau_{31}} + \frac{1}{\tau_{32}}\right) & 0 \\ 0 & u\eta_p I_p \sigma_{24} & 0 & -\frac{1}{\tau_{42}} \end{pmatrix}, \tag{2}$$

and the transposed form of electron population density matrix vector $\vec{N}_i$,

$$N_i^T = [\,N_1 \quad N_2 \quad N_3 \quad N_4\,], \tag{3}$$

are related to the total Er^{3+} dopant concentration N, through the conservation equation

$$N = \sum_{i=1}^{4} N_i, \tag{4}$$

where σ_{ij} denote absorption and emission cross-sections, $I_{p,s}$ photon intensities, $\eta_{p,s}$ proportions of power propagating within the core, and τ_{ij}^{-1} the spontaneous emission rates both radiative and non-radiative associated with the transition of electrons between relevant i and j bands with subscripts p and s for pump and signal radiations, respectively. u is the group velocity of light inside the fiber; $u = +1$ when the signal radiation travels in co-propagating direction, and $u = -1$ when it travels in counter-propagating direction with respect to the pump radiation along the longitudinal coordinate z.

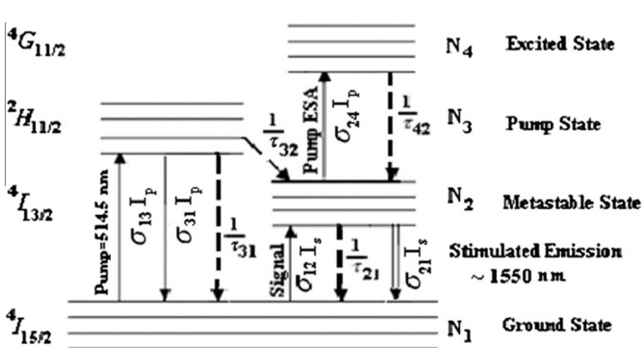

Fig. 1. Energy band scheme of Er^{3+} ions in glass matrix.

In the steady-state regime, the electron population densities are time invariant, $dN_i/dt = 0$. The solution of the system of transition rate equations (Eq. (1)) gives steady state population densities at individual levels in terms of pump and signal intensities as

$$N_i = \frac{N}{(\tau_{31}+\tau_{32})\{1+u\eta_s I_s \tau_{21}(\sigma_{12}+\sigma_{21})\}(1+CI_pDI_p^2)} \begin{pmatrix} (1+u\eta_s I_s \sigma_{21}\tau_{21})(\tau_{31}+\tau_{32}+u\eta_p I_p \sigma_{31}\tau_{31}\tau_{32}) \\ u\eta_s I_s \sigma_{12}\tau_{21}(\tau_{31}+\tau_{32}+u\eta_p I_p \sigma_{31}\tau_{31}\tau_{32}) + u\eta_p I_p \sigma_{13}\tau_{31}\tau_{21} \\ u\eta_p I_p \sigma_{13}\tau_{31}\tau_{32}(1+u\eta_s I_s \sigma_{21}\tau_{21}) \\ u\eta_p I_p \sigma_{24}\tau_{21}\tau_{42}\{u\eta_s I_s \sigma_{12}(\tau_{31}+\tau_{32}+u\eta_p I_p \sigma_{31}\tau_{31}\tau_{32}) + u\eta_p I_p \sigma_{13}\sigma_{31}\tau_{31}\} \end{pmatrix}, \tag{5}$$

where

$$C = \frac{u\eta_p[\tau_{31}\tau_{32}(1+u\eta_s I_s \sigma_{21}\tau_{21})(\sigma_{13}+\sigma_{31}) + \sigma_{13}\tau_{31}\tau_{21} + u\eta_s I_s \sigma_{12}\tau_{21}\{\sigma_{31}\tau_{31}\tau_{32}+\sigma_{24}\tau_{42}(\tau_{31}+\tau_{32})\}]}{(\tau_{31}+\tau_{32})\{1+u\eta_s I_s \tau_{21}(\sigma_{12}+\sigma_{21})\}}, \tag{6}$$

and