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Error estimates for discontinuous Galerkin method for nonlinear parabolic equations

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Abstract

We consider the nonlinear parabolic partial differential equations. We construct a discontinuous Galerkin approximation using a penalty term and obtain an optimal $L^\infty(L^2)$ error estimate.
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Keywords: Discontinuous Galerkin method; Error estimate; Nonlinear parabolic equation

1. Introduction

Discontinuous Galerkin methods with interior penalties for elliptic and parabolic equations were introduced by several authors [1,2,7]. They generalized Nitsche method in [3] to treat the Dirichlet boundary condition with penalty terms on the boundary of the domain. These methods referred to as interior penalty Galerkin schemes are not locally mass conservative.

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A new type of elementwise conservative discontinuous Galerkin method for diffusion problem was introduced and analyzed by Oden et al. [4]. Recently, Riviere and Wheeler [5] introduced a locally conservative discontinuous Galerkin formulation for nonlinear parabolic equations and derived a priori $L^\infty(L^2)$ and $L^2(H^1)$ error estimates. However, the error estimate in the $L^\infty(L^2)$ norm is not optimal.

The objective of this paper is to obtain an optimal error estimate in the $L^\infty(L^2)$ norm, which improves the result of [5]. A model problem and some assumptions are introduced in Section 2. In Section 3, we describe the definitions and the formulation of the discontinuous Galerkin method. And the optimal error estimates are obtained in Section 4.

2. A model problem

Consider the following nonlinear parabolic partial differential equation:

$$u_t - \nabla \cdot \big(a(x,u)\nabla u\big) = f(x,u), \quad (x,t) \in \Omega \times (0,\ T], \tag{2.1}$$

with the boundary condition

$$a(x,u)\nabla u \cdot n = 0, \quad (x,t) \in \partial\Omega \times (0,T], \tag{2.2}$$

and the initial condition

$$u(x,0) = g(x), \quad x \in \Omega, \tag{2.3}$$

where Ω is a bounded convex domain in $\mathbf{R}^d$, $d = 1, 2$, and n is a unit outward normal vector to $\partial\Omega$.

Assume that the following conditions are satisfied.

1. For any bounded subset B of real numbers, there exist constants γ and γ^* such that
$$0 < \gamma \leqslant a(x,p) \leqslant \gamma^*, \quad 0 < \gamma \leqslant \frac{\partial}{\partial p} a(x,p) \leqslant \gamma^* \quad \text{for any } (x,p) \in \Omega \times B.$$
2. a and f are uniformly Lipschitz continuous with respect to their second variable.
3. The model problem has a unique solution satisfying the following regularity conditions:
$$u \in L^2\big([0,T], H^s(\Omega)\big), \quad u_t \in L^2\big([0,T], H^s(\Omega)\big), \quad \text{for } s \geqslant 2;$$
$$u_t \in L^\infty\big([0,T], L^\infty(\Omega)\big), \quad \nabla u \in L^\infty\big(\Omega \times [0,T]\big).$$

3. A discontinuous Galerkin method

Let $\mathcal{E}_h = \{E_1, E_2, \cdot, E_{N_h}\}$ be a subdivision of Ω, where E_j is a triangle or a quadrilateral. Let $h_j = \operatorname{diam}(E_j)$ be the diameter of E_j and $h = \max\{h_j\colon\ j = 1, 2, \ldots, N_h\}$. We denote the edges of the elements by $\{e_1, e_2, \ldots, e_{P_h}, e_{P_h+1}, \ldots, e_{M_h}\}$, where $e_k \subset \Omega$, $1 \leqslant k \leqslant P_h$, and $e_k \subset \partial\Omega$, $P_h + 1 \leqslant k \leqslant M_h$. For each edge e_k, $P_h + 1 \leqslant k \leqslant M_h$, we take n_k the unit outward normal vector to $\partial\Omega$. And if $e_k = \partial E_i \cap \partial E_j$ for $i < j$ and $1 \leqslant k \leqslant P_h$ then we take n_k the unit outward normal vector to E_i.

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