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Existence of multiple positive solutions for one-dimensional p-Laplacian ☆

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Abstract

In this paper we consider the multipoint boundary value problem for one-dimensional p-Laplacian

$$\big(\phi_p(u')\big)' + f(t,u) = 0, \quad t \in (0,1),$$

subject to the boundary value conditions:

$$\phi_p\big(u'(0)\big) = \sum_{i=1}^{n-2} a_i \phi_p\big(u'(\xi_i)\big), \qquad u(1) = \sum_{i=1}^{n-2} b_i u(\xi_i),$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\xi_i \in (0,1)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{n-2} < 1$, and a_i, b_i satisfy $a_i, b_i \in [0,\infty]$, $0 < \sum_{i=1}^{n-2} a_i < 1$, and $\sum_{i=1}^{n-2} b_i < 1$. Using a fixed point theorem for operators on a cone, we provide sufficient conditions for the existence of multiple (at least three) positive solutions to the above boundary value problem.

Keywords: Multiple positive solutions; Boundary value problems; One-dimensional p-Laplacian

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1. Introduction

In this paper we study the existence of multiple positive solutions to the boundary value problem (BVP) for the one-dimensional p-Laplacian

$$\big(\phi_p(u')\big)' + f(t,u) = 0, \quad t \in (0,1), \tag{1.1}$$

$$\phi_p\big(u'(0)\big) = \sum_{i=1}^{n-2} a_i \phi_p\big(u'(\xi_i)\big), \qquad u(1) = \sum_{i=1}^{n-2} b_i u(\xi_i), \tag{1.2}$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\xi_i \in (0,1)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{n-2} < 1$ and a_i, b_i, f satisfy

(H_1) $a_i, b_i \in [0,\infty]$ satisfy $0 < \sum_{i=1}^{n-2} a_i < 1$, and $\sum_{i=1}^{n-2} b_i < 1$;
(H_2) $f \in C([0,1] \times [0,\infty), [0,\infty))$.

The study of multipoint boundary value problems for linear second-order ordinary differential equations was initiated by Il'in and Moiseev [1]. Since then there has been much current attention focused on the study of nonlinear multipoint boundary value problems, see [2–4,7].

In recent papers [5,6] the authors have investigated the following BVP for one-dimensional p-Laplacian:

$$\big(\phi_p(u')\big)' + a(t) f(t,u) = 0, \quad t \in (0,1), \tag{1.3}$$

$$u(0) = 0, \qquad u(1) = \sum_{i=1}^{m-2} a_i u(\xi_i), \tag{1.4}$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\xi_i \in (0,1)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{n-2} < 1$ and $\sum_{i=1}^{m-2} a_i \xi_i < 1$, and

$$\big(\phi_p(u')\big)' + f(t,u) = 0, \quad t \in (0,1), \tag{1.5}$$

$$u'(0) = \sum_{i=1}^{m-2} b_i u'(\xi_i), \qquad u(1) = \sum_{i=1}^{m-2} a_i u(\xi_i), \tag{1.6}$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\xi_i \in (0,1)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{n-2} < 1$ and a_i, b_i satisfy $0 < \sum_{i=1}^{m-2} a_i < 1$, $\sum_{i=1}^{m-2} b_i < 1$.

In paper [5] the authors claim that:

> It is easy to check that system (1.3) and (1.4) has a solution $u = u(t)$ if and only if u solves the operator equation
>
> $$u(t) = -\int_0^t \phi_q\left(\int_0^s a(\tau) f\big(\tau, u(\tau)\big)\, d\tau\right) ds - t\,\frac{\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} \phi_q(\int_0^s a(\tau) f(\tau, u(\tau))\, d\tau)\, ds}{1 - \sum_{i=1}^{m-2} a_i \xi_i}$$