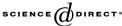
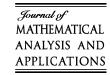


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Existence of multiple positive solutions for one-dimensional p-Laplacian $^{\Leftrightarrow}$

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Abstract

In this paper we consider the multipoint boundary value problem for one-dimensional p-Laplacian

$$(\phi_D(u'))' + f(t, u) = 0, \quad t \in (0, 1),$$

subject to the boundary value conditions:

$$\phi_p(u'(0)) = \sum_{i=1}^{n-2} a_i \phi_p(u'(\xi_i)), \qquad u(1) = \sum_{i=1}^{n-2} b_i u(\xi_i),$$

where $\phi_p(s) = |s|^{p-2}s$, p > 1, $\xi_i \in (0,1)$ with $0 < \xi_1 < \xi_2 < \dots < \xi_{n-2} < 1$, and a_i, b_i satisfy $a_i, b_i \in [0,\infty]$, $0 < \sum_{i=1}^{n-2} a_i < 1$, and $\sum_{i=1}^{n-2} b_i < 1$. Using a fixed point theorem for operators on a cone, we provide sufficient conditions for the existence of multiple (at least three) positive solutions to the above boundary value problem.

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Keywords: Multiple positive solutions; Boundary value problems; One-dimensional p-Laplacian

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1. Introduction

In this paper we study the existence of multiple positive solutions to the boundary value problem (BVP) for the one-dimensional p-Laplacian

$$\left(\phi_p(u')\right)' + f(t, u) = 0, \quad t \in (0, 1),$$
 (1.1)

$$\phi_p(u'(0)) = \sum_{i=1}^{n-2} a_i \phi_p(u'(\xi_i)), \qquad u(1) = \sum_{i=1}^{n-2} b_i u(\xi_i), \tag{1.2}$$

where $\phi_p(s) = |s|^{p-2}s$, p > 1, $\xi_i \in (0, 1)$ with $0 < \xi_1 < \xi_2 < \dots < \xi_{n-2} < 1$ and a_i, b_i, f satisfy

$$(H_1)$$
 $a_i, b_i \in [0, \infty]$ satisfy $0 < \sum_{i=1}^{n-2} a_i < 1$, and $\sum_{i=1}^{n-2} b_i < 1$; (H_2) $f \in C([0, 1] \times [0, \infty), [0, \infty))$.

The study of multipoint boundary value problems for linear second-order ordinary differential equations was initiated by Il'in and Moiseev [1]. Since then there has been much current attention focused on the study of nonlinear multipoint boundary value problems, see [2-4,7].

In recent papers [5,6] the authors have investigated the following BVP for one-dimensional *p*-Laplacian:

$$\left(\phi_{p}(u')\right)' + a(t)f(t,u) = 0, \quad t \in (0,1),$$
 (1.3)

$$u(0) = 0,$$
 $u(1) = \sum_{i=1}^{m-2} a_i u(\xi_i),$ (1.4)

where $\phi_p(s) = |s|^{p-2}s$, p > 1, $\xi_i \in (0, 1)$ with $0 < \xi_1 < \xi_2 < \dots < \xi_{n-2} < 1$ and $\sum_{i=1}^{m-2} a_i \xi_i < 1$, and

$$(\phi_n(u'))' + f(t, u) = 0, \quad t \in (0, 1),$$
 (1.5)

$$u'(0) = \sum_{i=1}^{m-2} b_i u'(\xi_i), \qquad u(1) = \sum_{i=1}^{m-2} a_i u(\xi_i), \tag{1.6}$$

where $\phi_p(s) = |s|^{p-2}s$, p > 1, $\xi_i \in (0, 1)$ with $0 < \xi_1 < \xi_2 < \dots < \xi_{n-2} < 1$ and a_i, b_i satisfy $0 < \sum_{i=1}^{m-2} a_i < 1$, $\sum_{i=1}^{m-2} b_i < 1$. In paper [5] the authors claim that:

It is easy to check that system (1.3) and (1.4) has a solution u = u(t) if and only if u solves the operator equation

$$u(t) = -\int_{0}^{t} \phi_{q} \left(\int_{0}^{s} a(\tau) f(\tau, u(\tau)) d\tau \right) ds$$
$$-t \frac{\sum_{i=1}^{m-2} a_{i} \int_{0}^{\xi_{i}} \phi_{q} (\int_{0}^{s} a(\tau) f(\tau, u(\tau)) d\tau) ds}{1 - \sum_{i=1}^{m-2} a_{i} \xi_{i}}$$

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