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Iterative procedures for solutions of random operator equations in Banach spaces

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Abstract

We construct random iterative processes for weakly contractive and asymptotically nonexpansive random operators and study necessary conditions for the convergence of these processes. It is shown that they converge to the random fixed points of these operators in the setting of Banach spaces. We also proved that an implicit random iterative process converges to the common random fixed point of a finite family of asymptotically quasi-nonexpansive random operators in uniformly convex Banach spaces.

Keywords: Random Mann iteration; Random Ishikawa iteration; Random operator; Three step random iterative process; Implicit random iterative process; Banach spaces; Measurable spaces

1. Introduction

Random nonlinear analysis is an important mathematical discipline which is mainly concerned with the study of random nonlinear operators and their properties and is much needed for the study of various classes of random equations. Random techniques have been

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crucial in diverse areas from pure mathematics to applied sciences. Of course famously random methods have revolutionised the financial markets. Random fixed point theorems for random contraction mappings on separable complete metric spaces were first proved by Spacek [24] and Hans [13,14]. The survey article by Bharucha-Reid [8] in 1976 attracted the attention of several mathematician and gave wings to this theory. Itoh [17] extended Spacek's and Hans's theorem to multivalued contraction mappings. Now this theory has become the full fledged research area and various ideas associated with random fixed point theory are used to obtain the solution of nonlinear random system (see [7]). Recently Papageorgiou [22], Xu [25], Beg [3–5], Xu and Beg [27], Liu [18], Beg and Shahzad [6] and many other authors have studied the fixed points of random maps. Choudhury [9], Mann [19], Outlaw [20], Ishikawa [15,16], Ghosh and Debnath [12], Park [21], Schu [23] and Choudhury and Ray [10] had used different iteration processes to obtain fixed points in deterministic operator theory. The aim of this paper is to study the different random iterative algorithms for weakly contractive and asymptotically nonexpansive random operators on an arbitrary Banach space. We also established the convergence of an implicit random iterative process for a finite family of asymptotically quasi-nonexpansive random operators.

2. Preliminaries

Let (Ω, Σ) be a measurable space (Σ-sigma algebra) and F a nonempty subset of a Banach space X. A mapping $\xi : \Omega \to X$ is *measurable* if $\xi^{-1}(U) \in \Sigma$ for each open subset U of X. The mapping $T : \Omega \times F \to F$ is a *random map* if and only if for each fixed $x \in F$, the mapping $T(., x) : \Omega \to F$ is measurable, and it is *continuous* if for each $\omega \in \Omega$, the mapping $T(\omega, .) : F \to X$ is continuous. A measurable mapping $\xi : \Omega \to X$ is the *random fixed point* of the random map $T : \Omega \times F \to X$ if and only if $T(\omega, \xi(\omega)) = \xi(\omega)$, for each $\omega \in \Omega$.

Let $B(x_0, r)$ denotes the spherical ball centered at x_0 with radius r, defined as the set $\{x \in X\colon \|x - x_0\| \leqslant r\}$.

We denote the nth iterate $T(\omega, T(\omega, T(\omega, \ldots, T(\omega, x))))$ of T by $T^n(\omega, x)$. The letter I denotes the random mapping $I : \Omega \times F \to F$ defined by $I(\omega, x) = x$ and $T^0 = I$.

Definition 2.1. Let F be a nonempty subset of a separable Banach space X and $T : \Omega \times F \to F$ be a random map. The map T is said to be:

(a) *Weakly contractive random operator* if for arbitrary $x, y \in F$,

$$\big\|T(\omega, x) - T(\omega, y)\big\| \leqslant \|x - y\| - \Psi\big(\|x - y\|\big), \quad \text{for each } \omega \in \Omega,$$

where $\Psi : [0, \infty) \to [0, \infty)$ is a continuous and nondecreasing map such that $\Psi(0) = 0$, and $\lim_{t\to\infty} \Psi(t) = \infty$.

(b) *Asymptotically contractive random operator* if for each $x \in F$, there exists x_0 in F with

$$\limsup_{n\to\infty} \frac{\|T(\omega, x) - T(\omega, x_0)\|}{\|x - x_0\|} < 1, \quad \text{for each } \omega \in \Omega.$$