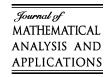


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Relaxed elastic line on a curved pseudo-hypersurface in pseudo-Euclidean spaces

A. Ceylan Çöken ^{a,*}, Ahmet Yücesan ^a, Nihat Ayyildiz ^a, Gerald S. Manning ^b

^a Süleyman Demirel University, Department of Mathematics, 32260, Isparta, Turkey ^b Rutgers University, Department of Chemistry and Chemical Biology, New Brunswick, NJ 08903, USA

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Abstract

In this work, we derive the Euler–Lagrange equation for an elastic line which is lying on a pseudo-hypersurface in pseudo-Euclidean spaces E^n_{ν} . Following this, we check the solutions which depend on the boundary conditions whether they are geodesic on a pseudo-hypersurface or not. The relaxed elastic line on a pseudo-hyperplane, a pseudo-hypersphere, and pseudo-hyperbolic space is a geodesic. However, the relaxed elastic line on a pseudo-hypercylinder, is a space-like geodesic. © 2005 Elsevier Inc. All rights reserved.

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E-mail addresses: ceylan@fef.sdu.edu.tr (A.C. Çöken), yucesan@fef.sdu.edu.tr (A. Yücesan), ayyildiz@fef.sdu.edu.tr (N. Ayyildiz), gmanning@rutchem.rutgers.edu (G.S. Manning).

^{*} Corresponding author.

1. Introduction

Let E_{ν}^{n} be *n*-dimensional pseudo-Euclidean space of signature $(\underbrace{+,\cdots,+}_{n-\nu},\underbrace{-,\cdots,-}_{\nu})$.

The metric tensor is given by

$$ds^{2} = \sum_{i=1}^{n-\nu} dx_{i}^{2} - \sum_{i=n+1-\nu}^{n} dx_{i}^{2},$$
(1.1)

where (x_1, \ldots, x_n) is a rectangular coordinate system of $\mathsf{E}^n_{\mathsf{v}}$ [6].

Definition 1.1. Let $n \ge 2$ and $0 \le \nu \le n$. Then,

(1) the pseudo-hypersphere of radius r > 0 in E_{ν}^{n} is the hyperquadric

$$S_{\nu}^{n-1}(r) = q^{-1}(r^2) = \{ p \in \mathsf{E}_{\nu}^n : \langle p, p \rangle = r^2 \},$$

with dimension n and index ν ;

(2) the pseudo-hyperbolic space of radius r > 0 in \mathbb{E}_{ν}^{n} is the hyperquadric

$$H^{n-1}_{\nu-1}(r) = q^{-1}(-r^2) = \left\{ p \in \mathsf{E}^n_{\nu} \colon \langle p, p \rangle = -r^2 \right\},\,$$

with dimension n and index ν [5].

An elastic line of length ℓ is defined as a curve with associated energy

$$K = \int_{0}^{\ell} k_1^2 \, ds,\tag{1.2}$$

where s is the arc length along the curve and k_1^2 is the first square curvature. The integral K is called the total square curvature.

If no boundary conditions are imposed at $s = \ell$, and if no external forces act at any s, the elastic line is *relaxed*. The trajectory of relaxed elastic line in space or on a pseudo-hyperplane is a straight line because the positive indefinite quantity that defines K takes its minimum value of zero when the first square curvature vanishes for all s. The trajectory of a relaxed elastic line constrained to lie on a general pseudo-hypersurface is, however, dependent on the intrinsic curvature of the pseudo-hypersurface, which in general bounds the possible values of K away from zero.

2. Preliminaries

Let α denote a curve on a connected oriented pseudo-hypersurface M in pseudo-Euclidean spaces E^n_{ν} . At a point $\alpha(s)$ of α , let $E_1 = \alpha'(s)$ denote the unit tangent vector.

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