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J. Math. Anal. Appl. 315 (2006) 367–378

Journal of
MATHEMATICAL
ANALYSIS AND
APPLICATIONS

www.elsevier.com/locate/jmaa

Relaxed elastic line on a curved pseudo-hypersurface in pseudo-Euclidean spaces

A. Ceylan Çöken [a,*], Ahmet Yücesan [a], Nihat Ayyildiz [a], Gerald S. Manning [b]

[a] *Süleyman Demirel University, Department of Mathematics, 32260, Isparta, Turkey*
[b] *Rutgers University, Department of Chemistry and Chemical Biology, New Brunswick, NJ 08903, USA*

Received 25 May 2004

Available online 11 July 2005

Submitted by H.R. Parks

Abstract

In this work, we derive the Euler–Lagrange equation for an elastic line which is lying on a pseudo-hypersurface in pseudo-Euclidean spaces E^n_ν. Following this, we check the solutions which depend on the boundary conditions whether they are geodesic on a pseudo-hypersurface or not. The relaxed elastic line on a pseudo-hyperplane, a pseudo-hypersphere, and pseudo-hyperbolic space is a geodesic. However, the relaxed elastic line on a pseudo-hypercylinder, is a space-like geodesic.

Keywords: Relaxed elastic line; Elasticity problem; Pseudo-Euclidean spaces; Pseudo-hypersurfaces; Geodesics; Euler–Lagrange equations

* Corresponding author.
E-mail addresses: ceylan@fef.sdu.edu.tr (A.C. Çöken), yucesan@fef.sdu.edu.tr (A. Yücesan), ayyildiz@fef.sdu.edu.tr (N. Ayyildiz), gmanning@rutchem.rutgers.edu (G.S. Manning).

doi:10.1016/j.jmaa.2005.05.051

1. Introduction

Let E^n_ν be n-dimensional pseudo-Euclidean space of signature $(\underbrace{+,\cdots,+}_{n-\nu},\underbrace{-,\cdots,-}_{\nu})$. The metric tensor is given by

$$ds^2=\sum_{i=1}^{n-\nu}dx_i^2-\sum_{i=n+1-\nu}^{n}dx_i^2, \tag{1.1}$$

where $(x_1,\ldots,x_n)$ is a rectangular coordinate system of E^n_ν [6].

Definition 1.1. Let $n\geqslant 2$ and $0\leqslant\nu\leqslant n$. Then,

(1) the pseudo-hypersphere of radius $r>0$ in E^n_ν is the hyperquadric

$$S^{n-1}_\nu(r)=q^{-1}(r^2)=\{p\in\mathsf{E}^n_\nu:\ \langle p,p\rangle=r^2\},$$

with dimension n and index ν;

(2) the pseudo-hyperbolic space of radius $r>0$ in E^n_ν is the hyperquadric

$$H^{n-1}_{\nu-1}(r)=q^{-1}(-r^2)=\{p\in\mathsf{E}^n_\nu:\ \langle p,p\rangle=-r^2\},$$

with dimension n and index ν [5].

An elastic line of length ℓ is defined as a curve with associated energy

$$K=\int_0^\ell k_1^2\,ds, \tag{1.2}$$

where s is the arc length along the curve and k_1^2 is the first square curvature. The integral K is called *the total square curvature*.

If no boundary conditions are imposed at $s=\ell$, and if no external forces act at any s, the elastic line is *relaxed*. The trajectory of relaxed elastic line in space or on a pseudo-hyperplane is a straight line because the positive indefinite quantity that defines K takes its minimum value of zero when the first square curvature vanishes for all s. The trajectory of a relaxed elastic line constrained to lie on a general pseudo-hypersurface is, however, dependent on the intrinsic curvature of the pseudo-hypersurface, which in general bounds the possible values of K away from zero.

2. Preliminaries

Let α denote a curve on a connected oriented pseudo-hypersurface M in pseudo-Euclidean spaces E^n_ν. At a point $\alpha(s)$ of α, let $E_1=\alpha'(s)$ denote the unit tangent vector.