

Lipschitz continuity theorems for free discontinuity problem in several variables[☆]

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Abstract

In this paper, following the method in [S. Solimini, Simplified excision techniques for free discontinuity problems in several variables, J. Funct. Anal. 151 (1997) 1–34], we prove a regularity of the function in minimizer for free discontinuity problem. Namely, we prove that the function is globally Lipschitz continuous out of a small neighborhood of the singular set.

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1. Introduction

In [2], S. Solimini by a simplified excision method, proved in arbitrary dimension, the uniform density property, the projection property and the bisection property. To prove these properties, he proved that the function in minimizer is globally Hölder continuous out of a small neighborhood of the singular set.

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In this paper, following the method in [2], we prove the regularity of the function. Namely we prove that the function is globally Lipschitz continuous out of a small neighborhood of the singular set.

2. Notions and statement of the results

We shall denote by Ω a given open subset of R^N and by g a given measurable function from Ω to $[0, 1]$. We shall denote by $H^\alpha(X)$ the Hausdorff measure of dimension α of a set X and by $|X|$ its Lebesgue measure. If u is any function defined on a subset of R^N , we shall denote by $\text{supp } u$ the support of u , namely the closure of the set where u takes values different from zero, and by oscillation of u on a subset X , namely $\text{osc}_X u = \sup_X u - \inf_X u$.

Given a closed subset K of Ω and a function $u \in L^1_{\text{loc}}(\Omega)$, we shall consider the functional

$$E(u, K) = H^{N-1}(K) + J(u, K),$$

where $J(u, K)$ denotes the value of the elliptic functional

$$J(u, K) = \int_{\Omega \setminus K} |\nabla u|^2 + |g - u|^2. \quad (1)$$

We shall always assume that K is a negligible set for the Lebesgue measure because, if this is not the case, the functional E takes a value equal to $+\infty$ on (u, K) , whatever u is, and therefore the set is not interesting in view of minimizing E . When the set variable K is given, one can determine a unique function u which minimizes the functional J on the open set $\Omega \setminus K$. Then one can consider the function u almost everywhere defined on the whole Ω , by assuming that K is negligible for the reason specified above, set

$$J(u, K) = \inf_{u \in H^1(\Omega \setminus K)} J(u, K),$$

$$E(u, K) = H^{N-1}(K) + J(u, K).$$

The function u takes values in $[0, 1]$.

When u and K are given and A is any open subset of Ω , we shall denote by $I_A(u)$ the value of $J_{A \setminus K}(u)$ and by $I(u)$ the value of $J(u, K)$. An easy estimate, which can be directly deduced from the definition of E , when (K, u) is a local minimum of E , states that for every open set $A \subset \Omega$

$$I_A(u) \leq |A|. \quad (2)$$

Indeed, if one can find a set A such that (2) does not hold, one can set $K' = K \setminus A$, and $u' = u$ on $\Omega \setminus A$, $u' = 0$ on A , then

$$\begin{aligned} E(u', K') &= E(u, K) - H^{N-1}(K \cap A) - I_A(u) + \int_A g^2 dx \\ &\leq E(u, K) - H^{N-1}(K \cap A) - I_A(u) + |A| < E(u, K), \end{aligned}$$

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