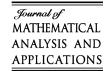




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Absolute equal distribution of the eigenvalues of discrete Sturm–Liouville problems

William F. Trench*

Trinity University, San Antonio, TX 78212, USA
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Abstract

We consider the asymptotic relationship as $n \to \infty$ between the eigenvalues $\lambda_{1n} \le \cdots \le \lambda_{nn}$ and $\mu_{1n} \le \cdots \le \mu_{nn}$ of the Sturm-Liouville problems defined for $n \ge 2k + 1$ by

$$\sum_{\ell=0}^{k} (-1)^{\ell} \Delta^{\ell} \left(r_{\ell n} (i - \ell) \Delta^{\ell} x_{i-\ell} \right) = \lambda \phi_{in} x_{i}, \quad 1 \leqslant i \leqslant n,$$

and

$$\sum_{\ell=0}^{k} (-1)^{\ell} \Delta^{\ell} \left(s_{\ell n} (i - \ell) \Delta^{\ell} x_{i-\ell} \right) = \mu \psi_{in} x_{i}, \quad 1 \leqslant i \leqslant n,$$

where $x_i = 0$ if $-k + 1 \le i \le 0$ or $n + 1 \le i \le n + k$, all quantities are real, and $\phi_{in}, \psi_{in} > 0, 1 \le i \le n$, $n \ge 2k + 1$. We give conditions implying that

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left| F(\lambda_{in}) - F(\mu_{in}) \right| = 0$$

for all $F \in C(-\infty, \infty)$ such that $\lim_{x \to -\infty} F(x)$ and $\lim_{x \to \infty} F(x)$ exist (finite). © 2005 Elsevier Inc. All rights reserved.

Keywords: Absolute equal distribution; Boundary conditions; Eigenvalue; Sturm-Liouville

^{*} Mailing address: 95 Pine Lane, Woodland Park, Colorado 80863, USA. E-mail address: wtrench@trinity.edu.

1. Introduction

We consider the asymptotic relationship as $n \to \infty$ between the eigenvalues of the Sturm–Liouville problems defined for $n \ge 2k + 1$ by

$$\sum_{\ell=0}^{k} (-1)^{\ell} \Delta^{\ell} \left(r_{\ell n} (i - \ell) \Delta^{\ell} x_{i-\ell} \right) = \lambda \phi_{in} x_{i}, \quad 1 \leqslant i \leqslant n, \tag{1}$$

and

$$\sum_{\ell=0}^{k} (-1)^{\ell} \Delta^{\ell} \left(s_{\ell n} (i - \ell) \Delta^{\ell} x_{i-\ell} \right) = \mu \psi_{in} x_i, \quad 1 \leqslant i \leqslant n, \tag{2}$$

with boundary conditions

$$x_i = 0 \quad \text{if } -k+1 \leqslant i \leqslant 0 \text{ or } n+1 \leqslant i \leqslant n+k. \tag{3}$$

We assume throughout that all quantities are real and ϕ_{in} , $\psi_{in} > 0$, $1 \le i \le n$, $n \ge 2k + 1$. If $\{d_n\}$ is a sequence of positive numbers, we write $a_n = o(d_n)$ to indicate that $\lim_{n \to \infty} a_n/d_n = 0$ and $a_n = O(d_n)$ to indicate that $\limsup_{n \to \infty} |a_n|/d_n < \infty$.

Definition 1. Let #S denote the cardinality of a set S. We say that a family of ordered sets $\{\{c_{in}\}_{i=i_0}^n\}_{n=2k+1}^{\infty}$ is essentially null if

$$\#\{i \mid i_0 \leqslant i \leqslant n \text{ and } |c_{in}| > \epsilon\} = o(n)$$

for every $\epsilon > 0$. We say that $\{\{c_{in}\}_{i=i_0}^n\}_{n=2k+1}^{\infty}$ is essentially bounded if there is an $M \in [0, \infty)$ such that

$$\#\{i \mid i_0 \leqslant i \leqslant n \text{ and } |c_{in}| > M\} = o(n).$$

This definition is closely related to [3, Definitions 4.2–4.4].

Let \mathcal{F} be the set of all $F \in C(-\infty, \infty)$ such that

$$\lim_{x \to -\infty} F(x) \quad \text{and} \quad \lim_{x \to \infty} F(x) \quad \text{exist (finite)}. \tag{4}$$

Let $\lambda_{1n} \leq \lambda_{2n} \leq \cdots \leq \lambda_{nn}$ and $\mu_{1n} \leq \mu_{2n} \leq \cdots \leq \mu_{nn}$ be the eigenvalues of (1), (3) and (2), (3), respectively. The following theorems are proved in Section 2.

Theorem 1. If $\{\{\phi_{in}\}_{i=1}^n\}_{n=2k+1}^{\infty}$, $\{\{1/\phi_{in}\}_{i=1}^n\}_{n=2k+1}^{\infty}$, and

$$\left\{ \left\{ r_{\ell n}(i) \right\}_{i=-\ell+1}^{n} \right\}_{n=2k+1}^{\infty}, \quad 0 \leqslant \ell \leqslant k,$$

are essentially bounded and

$$\left\{ \left\{ \phi_{in} - \psi_{in} \right\}_{i=1}^{n} \right\}_{n=2k+1}^{\infty} \quad and \quad \left\{ \left\{ r_{\ell n}(i) - s_{\ell n}(i) \right\}_{i=-\ell+1}^{n} \right\}_{n=2k+1}^{\infty}, \quad 0 \leqslant \ell \leqslant k,$$

are essentially null, then

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left| F(\lambda_{in}) - F(\mu_{in}) \right| = 0 \quad \text{for all } F \in \mathcal{F}.$$
 (5)

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