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Optimal control problems for the equation of motion of membrane with strong viscosity

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Abstract

Optimal control problems are studied for the equation of membrane with strong viscosity. The Gâteaux differentiability of solution mapping on control variables is proved and the various types of necessary optimality conditions corresponding to the distributive and terminal values observations are established.

Keywords: Optimal control; Equation of membrane with strong viscosity; Gâteaux differentiability; Necessary optimality conditions

1. Introduction

We consider a freely flexible stretched film which is called a membrane. It is well known that the vibration of the longitudinal motion of a membrane is described by the following nonlinear equation:

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$$\frac{\partial^2 y}{\partial t^2} - \operatorname{div}\left(\frac{\nabla y}{\sqrt{1+|\nabla y|^2}}\right) = 0, \tag{1.1}$$

where y is the height of the membrane. It seems to be difficult to construct a solution of (1.1) in a Hilbert or reflexive Banach spaces not only for theoretical construction but also for any other applications. So some modified but more realistic model equations are proposed, and among them we consider the following equation with strong viscosity terms:

$$\frac{\partial^2 y}{\partial t^2} - \operatorname{div}\left(\frac{\nabla y}{\sqrt{1+|\nabla y|^2}}\right) - \mu\Delta\frac{\partial y}{\partial t} = f, \tag{1.2}$$

where $\mu > 0$ and f is a forcing function. Equation (1.2) is proposed in Kobayashi et al. [4] and the well-posedness of strongly regular solutions are studied by using the resolvent estimates of linearized operators in a modified Banach space. Recently in Hwang [3] the well-posedness of less regular solutions, called weak solutions of (1.2) is proved in the framework of the variational method in Dautray and Lions [1] under Dirichlet boundary conditions. The result enables us to study the optimal control problems associated with (1.2) in the standard manner due to the theory of Lions [5]. We also refer to Ha and Nakagiri [2] for the optimal control problems on second order semilinear equations.

In this paper we study the optimal control problems for the controlled equation

$$\frac{\partial^2 y(v)}{\partial t^2} - \operatorname{div}\left(\frac{\nabla y(v)}{\sqrt{1+|\nabla y(v)|^2}}\right) - \mu\Delta\frac{\partial y(v)}{\partial t} = f + Bv, \tag{1.3}$$

where B is a controller, v is a control and $y(v)$ denotes the state for a given $v \in \mathcal{U}$, $\mathcal{U}$ is a Hilbert space of control variables. Let $\mathcal{U}_{ad} \subset \mathcal{U}$ be an admissible set. We propose the quadratic cost functional $J(v)$ as studied in Lions [5] and in Lions and Magenes [6]. The purpose of this paper is to establish the necessary conditions of optimality for various observation cases. For this we prove the Gâteaux differentiability of the nonlinear mapping $v \to y(v)$, which is used to define the associate adjoint system. We want to emphasize that in the velocity's observation case, a first order Volterra integro-differential equation is utilized as a proper adjoint system in spite of the original system being described by the second order equation.

2. Preliminaries

Let Ω be an open bounded set of $\mathbf{R}^n$ with the smooth boundary Γ. We set $Q = (0,T) \times \Omega$, $\Sigma = (0,T) \times \Gamma$ for $T > 0$. We consider the following Dirichlet boundary value problem for the equation of motion of a membrane with strong viscosity:

$$\begin{cases} \frac{\partial^2 y}{\partial t^2} - \operatorname{div}\left(\frac{\nabla y}{\sqrt{1+|\nabla y|^2}}\right) - \mu\Delta\frac{\partial y}{\partial t} = f \quad \text{in } Q, \\ y = 0 \quad \text{on } \Sigma, \\ y(0,x) = y_0(x), \quad \frac{\partial y}{\partial t}(0,x) = y_1(x) \quad \text{in } \Omega, \end{cases} \tag{2.1}$$