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Global attractivity of a positive periodic solution for a nonautonomous stage structured population dynamics with time delay and diffusion ☆

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Abstract

By employing the continuation theorem of coincidence degree theory, the existence of a positive periodic solution for a nonautonomous stage structured population dynamics with time delay and diffusion is established. Further, by constructing a Lyapunov functional and using the result of the existence of positive periodic solution, the attractivity of a positive periodic solution for above system is obtained.

Keywords: Time delay; Stage-structures; Diffusion; Positive periodic solutions; Lyapunov functional; The continuation theorem of coincidence degree theory

1. Introduction

In the study of the population dynamics, in order to make the population models more practical and accurate, more and more realistic factors have been considered, such as stage-structure (see [1–4,12,13]), diffusion (see [2,5–7,13]), time delay (see [1,5–7,10,11]), but the models with all the factors seem to be rarely considered.

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To consider all these factors, Li et al. [8] introduced the following nonautonomous population model with stage-structure, diffusion and time delay:

$$\begin{cases} x_1'(t) = \alpha_1(t)y_1(t) - r_1(t)x_1(t) - \alpha_1(t-\tau)\exp\left(-\int_{t-\tau}^{t} r_1(s)\,ds\right)y_1(t-\tau), \\ y_1'(t) = \alpha_1(t-\tau)\exp\left(-\int_{t-\tau}^{t} r_1(s)\,ds\right)y_1(t-\tau) - \beta_1(t)y_1^2(t) \\ \qquad + D_1(t)\big(y_2(t) - y_1(t)\big) + R(t)y_1(t)z(t), \\ z'(t) = \alpha_3(t)z(t) - r_3(t)z^2(t) - \theta(t)y_1(t)z(t), \\ x_2'(t) = \alpha_2(t)y_2(t) - r_2(t)x_2(t) - \alpha_2(t-\tau)\exp\left(-\int_{t-\tau}^{t} r_2(s)\,ds\right)y_2(t-\tau), \\ y_2'(t) = \alpha_2(t-\tau)\exp\left(-\int_{t-\tau}^{t} r_2(s)\,ds\right)y_2(t-\tau) - \beta_2(t)y_2^2(t) + D_2(t)\big(y_1(t) - y_2(t)\big), \end{cases} \tag{1.1}$$

where $x_i(t)$, $y_i(t)$, $i = 1, 2$, represent the immature and mature predator population densities in the path i, respectively. $z(t)$ represents the prey population density in the patch 1, and $y_1(t)$, in patch 1, is its predator. The mature population $y_i(t)$, $i = 1, 2$, can disperse between the two patches. τ denotes the length of time that predator i $(i = 1, 2)$ grow from the birth to maturity, $\alpha_i(t)$ denotes the bearing rate of immature predator in patch i, $i = 1, 2$, $\alpha_i(t-\tau)$ denotes the bearing rate of mature predator in patch i, $i = 1, 2$, $r_i(t)$ is the death rate of immature predator in patch i, $i = 1, 2$, $\beta_i(t)$ denotes the death rate of mature predator in patch i, $i = 1, 2$, $R(t)$ denotes the preying effective rate in patch 1, $D_i(t)$ is the diffusive coefficients of mature predator in patch i, $i = 1, 2$, α_3 denotes the bearing rates of prey in patch 1, r_3 denotes the death rate of prey in patch 1, θ denotes the preying rate in patch 1, $\alpha_i(t)$, $r_i(t)$ $(i = 1, 2, 3)$, $\beta_i(t)$, $D_i(t)$ $(i = 1, 2)$, $R(t)$, $\theta(t)$ are positive continuous functions on $[0, +\infty)$.

In [8], the authors first established the existence of a positive periodic solution by using a fixed point theorem (see [8]) and the persistent result drawn by them. Then obtained the sufficient condition for a unique positive periodic solution which is globally attractive by using Lyapunov functional. To state the result in [8], we make two assumptions and a notation:

(H1) all the coefficients in system (1.1) are positive continuous ω-periodic functions with $\omega > 0$;

(H2) $0 < \min\{\underline{\alpha_i},\ \underline{r_i}\ (i = 1, 2, 3),\ \underline{\beta_i},\ \underline{D_i}\ (i = 1, 2),\ \underline{R},\ \underline{\theta}\}$
$\leqslant \max\{\overline{\alpha_i},\ \overline{r_i}\ (i = 1, 2, 3), \overline{\beta_i},\ \overline{D_i}\ (i = 1, 2),\ \overline{R},\ \bar{\theta}\} < +\infty.$

For a positive continuous ω-periodic function $f(t)$, we set

$$\overline{f} = \max_{t\in[0,\omega]}\{f(t)\}, \qquad \underline{f} = \min_{t\in[0,\omega]}\{f(t)\}, \qquad \tilde{f} = \frac{1}{\omega}\int_0^{\omega} f(t)\,dt.$$