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Original article

On micropolar elastic cusped prismatic shells

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Abstract

A huge literature is devoted to the study of cusped prismatic shells on the basis of the classical theory of elasticity. It was stimulated by the works of I. Vekua. I. Vekua considered very important to carry out investigations of boundary value and initial boundary value problems for such bodies, since they are connected with degenerate partial differential equations and, therefore, are not classical, in general. The present paper is devoted to cusped prismatic shells on the basis of the theory of micropolar elasticity. Namely, on the basis of the $N = 0$ approximation of hierarchical models for micropolar elastic cusped prismatic shells constructed by the I. Vekua dimension reduction method.

Keywords: Micropolar elasticity; Cusped prismatic shells; Degenerate partial differential equations

1. Introduction

A huge literature is devoted to the study of cusped prismatic shells on the basis of the classical theory of elasticity. It was stimulated by the work [1] of I. Vekua (see also [2]). I. Vekua considered very important to carry out investigations of boundary value (BVP) and initial boundary value (IBVP) problems for such bodies, since they are connected with degenerate partial differential equations (PDE) and, therefore, are not classical, in general. A survey of results obtained in this direction one can find in [3] (see also the references therein). The present paper is devoted to cusped prismatic shells on the basis of the theory of micropolar elasticity (see, e.g. [4,5]). Namely, on the basis of the $N = 0$ approximation of hierarchical models for micropolar elastic cusped prismatic shells constructed by the I. Vekua dimension reduction method.

The paper is organized as follows. In Section 2 we give an exposition of the governing equations of the $N = 0$ approximation of hierarchical models of micropolar elastic prismatic shells and briefly sketch the Nth approximation. Section 3 contains an analysis of the system of PDEs constructed in Section 2. Peculiarities of well-posedness of boundary conditions (BCs) for micropolar elastic cusped prismatic shells are revealed.

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2. $N = 0$ approximation

Let $Ox_1x_2x_3$ be an anticlockwise-oriented rectangular Cartesian frame of origin O. We conditionally assume the x_3-axis vertical. The elastic body Ω is called a prismatic shell [1–3] if it is bounded from above and below by, respectively, the surfaces (so called face surfaces)

$$x_3 = \overset{(+)}{h}(x_1, x_2) \quad \text{and} \quad x_3 = \overset{(-)}{h}(x_1, x_2), \quad (x_1, x_2) \in \omega,$$

laterally by a cylindrical surface Γ of generatrix parallel to the x_3-axis and its vertical dimension is sufficiently small compared with other dimensions of the body. $\overline{\omega} := \omega \cup \partial\omega$ is the so-called projection of the prismatic shell on $x_3 = 0$.

Let the thickness of the prismatic shell be

$$2h(x_1, x_2) := \overset{(+)}{h}(x_1, x_2) - \overset{(-)}{h}(x_1, x_2) \quad \begin{cases} > 0 & \text{for} \quad (x_1, x_2) \in \omega, \\ \geq 0 & \text{for} \quad (x_1, x_2) \in \partial\omega \end{cases}$$

and

$$2\widetilde{h}(x_1, x_2) := \overset{(+)}{h}(x_1, x_2) + \overset{(-)}{h}(x_1, x_2).$$

If the thickness of the prismatic shell vanishes on some subset of $\partial\omega$, it is called cusped one.

Let us note that the lateral boundary of the standard shell is orthogonal to the "middle surface" of the shell, while the lateral boundary of the prismatic shell is orthogonal to the prismatic shell's projection on $x_3 = 0$.

Let $t \in T := [0, +\infty[$ be time, $T_+ :=]0, \infty[$, $\overline{\Omega} \times T$ denote the Cartesian product, $u_i \in C^2(\Omega \times T_+)$, $i = 1, 2, 3$, be displacements, $\omega_i \in C^2(\Omega \times T_+)$, $i = 1, 2, 3$, be microrotations, $e_{ij} \in C^1(\Omega \times T_+)$ be the asymmetric strain tensor, $u_{ji} \in C^1(\Omega \times T_+)$ be the asymmetric microstrain (torsion-flexure) tensor, $X_{ji} \in C^1(\Omega \times T_+)$ be the asymmetric force–stress tensor, $\chi_{ji} \in C^1(\Omega \times T_+)$ be the asymmetric couple stress tensor, $\Phi_i \in C(\Omega \times T_+)$ and $\Psi_i \in C(\Omega \times T_+)$ be the fields of volume forces and volume couples, respectively, ρ be the density, $\mathcal{I}$ be the rotational inertia of the medium, λ, μ, $\widetilde{\alpha}$, $\widetilde{\beta}$, ν and ε be the elasticity constants of the medium, $\mu > 0$, $3\lambda + 2\mu > 0$, $\widetilde{\alpha} > 0$, $\widetilde{\beta} > 0$, $\nu > 0$, $3\varepsilon + 2\nu > 0$, $\in_{ijk}$ be the Levi-Civita symbol. Here C^2 and C^1 are classes of twice and once, correspondingly, continuously differentiable functions in the domain under consideration; C is a class of continuous functions on the sets under consideration. Throughout the paper Einstein's rule of summation is used for Latin indexes from 1 to 3, and for Greek indexes from 1 to 2.

In order to construct governing equations of the $N = 0$ approximation of hierarchical models, using Vekua's dimension reduction method, we integrate within the limits $\overset{(-)}{h}$, $\overset{(+)}{h}$ with respect to the thickness variable x_3 the following governing equations of the micropolar theory of elasticity (see [4,5] and the references therein):

Motion equations

$$X_{ji,j} + \Phi_i = \rho \ddot{u}_i, \qquad i = 1, 2, 3, \tag{1}$$

$$\chi_{ji,j} + \in_{ijk} X_{jk} + \Psi_i = \mathcal{I}\ddot{\omega}_i, \qquad i = 1, 2, 3; \tag{2}$$

Kinematic equations

$$u_{ji} = u_{i,j} - \in_{kji} \omega_k = e_{ji} + \in_{kji}(\theta_k - \omega_k), \quad i, j = 1, 2, 3, \tag{3}$$

$$\omega_{ji} = \omega_{i,j}, \quad i, j = 1, 2, 3; \tag{4}$$

Constitutive equations

$$\begin{aligned} X_{ij} &= \lambda\delta_{ij}u_{kk} + (\mu + \tilde{\alpha})u_{ij} + (\mu - \tilde{\alpha})u_{ji} = \lambda u_{k,k}\delta_{ij} \\ &\quad + (\mu + \tilde{\alpha})u_{j,i} - (\mu + \tilde{\alpha}) \in_{kij} \omega_k + (\mu - \tilde{\alpha})u_{i,j} - (\mu - \tilde{\alpha}) \in_{kji} \omega_k \\ &= \lambda u_{k,k}\delta_{ij} + (\mu + \tilde{\alpha})u_{j,i} + (\mu - \tilde{\alpha})u_{i,j} - 2\tilde{\alpha} \in_{kij} \omega_k, \quad i, j = 1, 2, 3, \end{aligned} \tag{5}$$

$$\begin{aligned} \chi_{ij} &= \varepsilon\delta_{ij}\omega_{kk} + (\nu + \tilde{\beta})\omega_{ij} + (\nu - \tilde{\beta})\omega_{ji} \\ &= \varepsilon\omega_{k,k}\delta_{ij} + (\nu + \tilde{\beta})\omega_{j,i} + (\nu - \tilde{\beta})\omega_{i,j}, \quad i, j = 1, 2, 3, \end{aligned} \tag{6}$$