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Original article

An approximate solution of one class of singular integro-differential equations

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Abstract

The problem of definition of mechanical field in a homogeneous plate supported by finite inhomogeneous inclusion is considered. The contact between the plate and inclusion is realized by a thin glue layer. The problem is reduced to the boundary value problem for singular integro-differential equations. Asymptotic analysis is carried out. Using the method of orthogonal polynomials, the problem is reduced to the solution of an infinite system of linear algebraic equations. The obtained system is investigated for regularity.

Keywords: Elastic inclusion; Integro-differential equation; Orthogonal polynomials

1. Statement of the Problem and its Reduction to a Singular Integro-Differential Equation (SIDE)

Let an elastic plane with the modulus of elasticity E_2 and the Poisson coefficient ν_2 on a finite interval $[-1, 1]$ of the ox-axis be reinforced by an inclusion in the form of a cover plate of small thickness $h_1(x)$, with the modulus of elasticity $E_1(x)$ and the Poisson coefficient ν_1, loaded by tangential force of intensity $\tau_0(x)$, and the plate at infinity towards to the ox and oy-axes be subjected to uniformly stretching forces of intensities p and q, respectively.

Under the conditions of plane deformation we are required to determine contact stresses acting in the interval of the inclusion and plate joint. An inclusion will be assumed to be a thin plate free from bending rigidity, and the contact between the plate and inclusion is realized by a thin glue layer with thickness h_0 and modulus of shear G_0.

Equation of equilibrium of differential element of inclusion has the form [1]

$$\frac{d}{dx}\left(E(x)\frac{du_1(x)}{dx}\right) = \tau_-(x) - \tau_+(x) - \tau_0(x), \quad |x| < 1, \tag{1}$$

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where $\tau_{\pm}(x)$ are unknown tangential contact stresses at the upper and lower contours of the inclusion, $u_1(x)$ is horizontal displacement of inclusion points towards the ox-axis, $E(x) = \frac{E_1(x)h_1(x)}{1-\nu_1^2}$. Introducing the notation $\tau(x) := \tau_-(x) - \tau_+(x)$ and based on Eq. (1), deformation of points of inclusion can be expressed as

$$\varepsilon_x^{(1)} := \frac{du_1(x)}{dx} = \frac{1}{E(x)} \int_{-1}^{x} [\tau(t) - \tau_0(t)]dt, \quad |x| < 1. \tag{2}$$

The condition of equilibrium of the inclusion has the form

$$\int_{-1}^{1} [\tau(t) - \tau_0(t)]dt = 0. \tag{3}$$

Assuming that every element of the glue layer is under the conditions of pure shear, the contact condition has the form [2]

$$u_1(x) - u_2(x, 0) = k_0\tau(x), \quad |x| \le 1, \tag{4}$$

where $u_2(x, y)$ are displacement of the plate points along the ox-axis, $k_0 := h_0/G_0$.

On the basis of the well-known results (see, e.g., [3]), the deformation $\varepsilon_x^{(2)} := \frac{du_2(x,0)}{dx}$ of the plane point along the ox-axis caused by the force factors $\tau(x)$, p and q is represented in the form

$$\varepsilon_x^{(2)} = \frac{\aleph}{2\pi\mu_2(1+\aleph)} \int_{-1}^{1} \frac{\tau(t)dt}{t-x} + \frac{\aleph+1}{8\mu_2}p + \frac{\aleph-3}{8\mu_2}q, \tag{5}$$

where $\aleph = 3 - 4\nu_2$, while λ_2 and μ_2 are the Lamé parameters.

Taking into account (2) and (5), from the contact conditions (4), we get

$$\frac{1}{E(x)} \int_{-1}^{x} [\tau(t)dt - \tau_0(t)]dt - \frac{\aleph}{2\pi\mu_2(1+\aleph)} \int_{-1}^{1} \frac{\tau(t)dt}{t-x} - \frac{\aleph+1}{8\mu_2}p - \frac{\aleph-3}{8\mu_2}q = k_0\tau'(x), \quad |x| < 1. \tag{6}$$

In the notations

$$\varphi(x) = \int_{-1}^{x} [\tau(t) - \tau_0(t)]dt, \quad \lambda = \frac{\aleph}{2\mu_2(1+\aleph)},$$
$$g(x) = \frac{\lambda}{\pi} \int_{-1}^{1} \frac{\tau_0(t)dt}{t-x} + k_0\tau_0'(x) + \frac{\aleph+1}{8\mu_2}p + \frac{\aleph-3}{8\mu_2}q,$$

we rewrite Eq. (6) in the form

$$\frac{\varphi(x)}{E(x)} - \frac{\lambda}{\pi} \int_{-1}^{1} \frac{\varphi'(t)dt}{t-x} - k_0\varphi''(x) = g(x), \quad |x| < 1. \tag{7}$$

Thus the equilibrium condition (3) takes the form

$$\varphi(1) = 0. \tag{8}$$

Thus the above posed boundary contact problem is reduced to the solution of SIDE (7) with the condition (8). From the symmetry of the problem, we assume, that function $E(x)$ is even and external load $\tau_0(x)$ is uneven, the solution of Eq. (7) under the condition (8) can be sought in the class of even functions. Moreover, we assume that the function is continuous and has a continuous first order derivative on the interval $[-1, 1]$.

2. Asymptotic investigation

Under the assumption that

$$E(x) = (1-x^2)^{\gamma} b_0(x), \quad \gamma \ge 0, \quad b_0(x) = b_0(-x), \quad b_0 \in C([-1, 1]), \tag{9}$$
$$b_0(x) \ge c_0 = \text{const} > 0$$