Available online at www.sciencedirect.com
ScienceDirect
Transactions of A. Razmadze Mathematical Institute 170 (2016) 208–211
Transactions of A. Razmadze Mathematical Institute
www.elsevier.com/locate/trmi

Original article

The Riemann–Hilbert problem in the class of Cauchy type integrals with densities of grand Lebesgue spaces

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Available online 25 July 2016

Abstract

The present paper deals with a solution of the Riemann–Hilbert problem in the class of Cauchy type integrals with densities of certain new nonstandard Banach function spaces. The solvability conditions are explored and the solutions (if any) are constructed explicitly.
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Keywords: Grand Lebesgue spaces; Riemann–Hilbert problem; Cauchy type integrals; Weights

1. Introduction

The grand Lebesgue spaces were introduced by T. Iwaniec and C. Sbordone in [1], where they studied the integrability problem of the Jacobian under minimal hypotheses. Later on, the more general Lebesgue grand spaces $L^{p),\theta}(1 < p < \infty, \theta > 0)$ appeared in the paper of L. Greco, T. Iwaniec and S. Sbordone [2] in which they studied the existence and uniqueness of solutions to the inhomogeneous n-harmonic equation div $A(x, \nabla u) = \mu$. The necessity to investigate these spaces has emerged from their rather essential role in various fields, in particular, in nonlinear partial differential equations. It turns our that the spaces $L^{p),\theta}$ are intended to establish the existence and uniqueness, as well as the regularity for various PDEs.

The boundedness in weighted grand Lebesgue spaces of fundamental integral operators in linear and nonlinear harmonic analysis is established in [3–6] (see also [7, Ch. 14] and [8, Ch. 2]).

It should be emphasized that the first author has established the necessary and sufficient conditions for the curve and the weight simultaneously ensuring the boundedness of the operator generated by the Cauchy singular integral

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Peer review under responsibility of Journal Transactions of A. Razmadze Mathematical Institute.

http://dx.doi.org/10.1016/j.trmi.2016.07.002
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defined on the rectifiable curve. The Dirichlet and Riemann boundary value problems in the framework of grand Lebesgue spaces are solved in [9] (see also [8, Ch. 4]).

In the present work, we present the solution of the Riemann–Hilbert problem

$$\mathrm{Re}[\lambda(t)\phi^{+}(t)] = b(t) \tag{1}$$

in the class $K^{p),\theta}(D)$, i.e., a set of the Cauchy type integrals

$$\phi(z) = \frac{1}{2\pi i}\int_{\Gamma}\frac{\varphi(t)}{t-z}dt, \quad z \in D,$$

where D is a simply-connected bounded domain with the boundary Γ and $\varphi \in L^{p),\theta}(\Gamma)$, $(1 < p < \infty, \theta > 0)$.

The definition of the grand Lebesgue spaces and the conditions for the boundary Γ and for the functions $\lambda(t)$ and $b(t)$ are given in the next section.

2. Preliminaries

Let Γ be a simple rectifiable curve. Suppose that ω is a weight function prescribed on Γ. The weighted grand Lebesgue space $L^{p),\theta}_{\omega}(\Gamma)$ $(1 < p < \infty, \theta > 0)$ is defined by the norm

$$\|f\|_{L^{p),\theta}_{\omega}(\Gamma)} = \sup_{0<\varepsilon<p-1}\left(\varepsilon^{\theta}\int_{\Gamma}|f(t)|^{p-\varepsilon}\omega(t)|dt|\right)^{\frac{1}{p-\varepsilon}}.$$

$L^{p),\theta}_{\omega}(\Gamma)$ is a Banach function space.

Let now D be a simply-connected bounded domain with the boundary Γ and let $z = z(w)$ be conformal mapping of a circle $U = \{w : |w| < 1\}$ onto D. By $w = w(z)$ we denote its inverse mapping. Assume $\gamma = \{\tau : |\tau| = 1\}$.

Here we introduce certain classes of analytic functions.

For $1 < p < \infty, \theta > 0$ we put:

$$K^{p),\theta}(D) = \left\{\phi : \phi(z) = (K_{\Gamma}\varphi)(z) = \frac{1}{2\pi i}\int_{\Gamma}\frac{\varphi(t)dt}{t-z}, \quad \varphi \in L^{p),\theta}(\Gamma), \quad z \in D\right\},$$

$$\widetilde{K}^{p),\theta}(\mathbb{C}\setminus\Gamma) = \left\{\phi : \phi(z) = (K_{\Gamma}\varphi)(z) + Q(z), \quad \varphi \in L^{p),\theta}(\Gamma), \quad z \in \mathbb{C}\setminus\Gamma \quad Q \text{ is a polynomial}\right\},$$

$$K^{p),\theta}_{\omega}(U) = \left\{F : F(w) = \frac{1}{2\pi i}\int_{\gamma}\frac{f(\tau)}{\tau-w}d\tau, \quad f \in L^{p),\theta}_{\omega}(\gamma), \quad w \in U\right\}$$

and

$$\widetilde{K}^{p),\theta}_{\omega}(\mathbb{C}\setminus\gamma) = \left\{F : F(w) = \frac{1}{2\pi i}\int_{\gamma}\frac{f(\tau)d\tau}{\tau-w} + q(w), \quad f \in L^{p),\theta}_{\omega}(\gamma), \quad w \in \mathbb{C}\setminus\gamma, \quad q \text{ is a polynomial of } w\right\}.$$

First of all, we adduce our assumptions for the curve Γ.

In what follows, it will be assumed that

$$1/z'(w) \in \bigcup_{\delta>0} H^{\delta}(U), \quad \text{and} \quad z'(\tau) \in A_p(\gamma)$$

where H^{δ} denotes a class of analytic Hardy class functions and $A_p(\gamma)$ is a class of weighted Muckenhoupt functions, i.e., a set of weight functions ω defined on γ for which

$$\sup\left(\frac{1}{|l|}\int_{l}\omega(\tau)|d\tau|\right)\left(\frac{1}{|l|}\int_{l}\omega^{1-p'}(\tau)|d\tau|\right)^{p-1} < +\infty,$$

where the least upper bound is taken over all arcs l of the unit circumference γ.

As for the coefficients and the right-hand side of (1), it is required that: $\lambda(t) \in C(\Gamma)$, $\lambda(t) \neq 0$ the real function $b \in L^{p),\theta}$, $a(t) = \overline{\lambda}(t)/\lambda(t)$, and the index $\varkappa = \mathrm{ind}_{\Gamma}\, a(t) = \frac{1}{2\pi}[\arg a(t)]_{\Gamma}$.

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