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Original article

The nonstationary flow of a conducting fluid in a plane pipe in the presence of a transverse magnetic field

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Abstract

We consider the nonstationary flow of an incompressible viscous conducting fluid in the plane pipe of infinite length in the presence of a transverse magnetic field. Using the Laplace transformation we obtain the expressions for the fluid flow velocity and the electric and magnetic field intensities when the conductivity values of the fluid and pipe walls are arbitrary. Solutions are expressed in terms of complex integrals which are calculated for the particular case of ideally conducting walls.

Keywords: Nonstationary flow; Viscous conducting fluid; Plane pipe; Damped oscillation

1. Introduction

In recent years, nonstationary flows of a conducting incompressible fluid have been considered in a number of works. A class of exact solutions of magnetohydrodynamic equations for laminar flows has been considered in the papers [1–3]. The theoretical statement of nonstationary problems and their solvability were investigated by Ladyzhenskaya and Solonnikov in [4]. In the papers [5–7], an exact solution was obtained for a nonstationary flow of a fluid which is produced by the ideally conducting parallel walls in the presence of a transverse magnetic field. The impulsive motion and oscillations of the plate in a conducting fluid in the presence of a magnetic field are studied in the works [8–12].

2. Main part

In the present paper, an exact solution is obtained for the particular case of a nonstationary flow of a conducting incompressible viscous fluid between the conducting parallel walls of infinite length. An analogous problem was the subject of Regirer's paper, but the induced fields outside the fluid are ignored there.

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Let a stationary fluid, whose conductivity is σ, viscosity coefficient is η, density is ρ and magnetic permeability is μ, occupy an infinitely long plane pipe with parallel walls, the distance between which is $2L$. The pipe walls are assumed to be infinitely thick and characterized by the conductivity σ^*, magnetic permeability μ and dielectric permeability ε. There exists a transverse magnetic field $B_0 = \mu H_0$.

At the initial moment of time $t = 0$, the constant pressure changes suddenly along the pipe and, as a result, the fluid begin to move. The origin of the Cartesian system (right) is chosen between the pipe walls, the x-axis coincides with the fluid motion direction, while the y-axis is directed normally to the walls, i.e. in parallel to the direction of the magnetic field.

To prevent the appearance of electric bulk charges, it is assumed that the conducting walls are grounded at $z \to \pm\infty$.

Let the values L, U_0, $\frac{L}{U_0}$, ρU_0^2, H_0, $\mu H_0 U_0$, $\frac{H_0}{L}$ (U_0 is some typical velocity) denote respectively the radius of the vector $\vec{r}$, fluid velocity $\vec{V}$, time t, pressure ρ, magnetic field intensity $\vec{H}$, electric field intensity $\vec{E}$ and current density $\vec{j}$.

Then the equations of the problem will be written in the non-dimensional form [13–15] as follows: in the domain adjacent to the fluid

$$\operatorname{rot} \vec{H} = \vec{j} \tag{1}$$

$$\operatorname{div} \vec{H} = 0 \tag{2}$$

$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{H}}{\partial t}, \tag{3}$$

$$\operatorname{div} \vec{E} = 0 \tag{4}$$

$$\vec{j} = R_m(\vec{E} + \vec{V} \times \vec{H}) \tag{5}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}\,\nabla)\vec{V} = -\nabla\rho + S(\operatorname{rot}\vec{H} \times \vec{H}) + \frac{1}{R}\Delta\vec{V} \tag{6}$$

$$\operatorname{div} \vec{V} = 0, \tag{7}$$

in the domain near the pipe walls

$$\operatorname{rot} \vec{H}^* = \vec{j}^* + \beta^2 \frac{\partial \vec{E}^*}{\partial t}, \tag{8}$$

$$\operatorname{div} \vec{H}^* = 0, \tag{9}$$

$$\operatorname{rot} \vec{E}^* = -\frac{\partial \vec{H}^*}{\partial t}, \tag{10}$$

$$\operatorname{div} \vec{E}^* = 0, \tag{11}$$

$$\vec{j}^* = R_m^* \vec{E}^*, \tag{12}$$

where

$$\left.\begin{aligned} S = \frac{B_0^2}{\mu\rho U_0^2} = \frac{M^2}{R R_m}, \qquad M^2 = \frac{B_0^2 L^2 \sigma}{\eta}, \qquad \beta^2 = \varepsilon\mu U_0^2, \\ R = \frac{U_0 L \rho}{\mu}, \qquad R_m = \sigma\mu U_0 L, \qquad R_m^* = \sigma^*\mu U_0 L, \end{aligned}\right\}$$

are non-dimensional parameters.

As it is usually done in magnetohydrodynamics, we neglect the displacement current in the fluid.

In the considered problem

$$\vec{V} = \vec{V}[U(y,t), 0, 0], \qquad \vec{H} = \vec{H}[H_x(y,t), 1, 0], \qquad \vec{E} = \vec{E}[0, 0, E_x(y,t)],$$