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Original article

Common fixed point results for weakly compatible mappings under contractive conditions of integral type in complex valued metric spaces

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Abstract

In this manuscript, using $(E.A)$ property and (CLR) property common fixed point results for weakly compatible mappings, satisfying integral type contractive condition in complex valued metric spaces are investigated.

Keywords: Complex valued metric spaces; Common fixed points; Weakly compatible mappings; Property $(E.A)$; (CLR) property

1. Introduction

Banach contraction principle [1] is the most powerful result in the field of metric fixed point theory. This principle provides distinctive solution to various mathematical models such as Integral equations, Differential equations and Functional equations. Banach's contraction principle has been extended and generalized for different kinds of contractions in various metric spaces. A significant generalization of Banach principle [1] is the Branciari [2] fixed point theorem for integral type inequality. Afterward, several researchers [3–8] further generalized the result of Branciari in metric spaces.

Azam et al. [9] introduced the notion of complex valued metric space and proved common fixed point theorems for two self-mappings satisfying a rational type inequality. Bhatt et al. [10] initiated the concept of weakly compatible maps to study common fixed point theorem for weakly compatible maps in complex valued metric spaces. Verma and Pathak [11] introduced the notion of property $(E.A)$ and (CLR) property and established common fixed point theorems using these properties in complex valued metric space. Manro et al. [12] generalized the theorem of Branciari [2] for two self-maps under contractive condition of integral type satisfying $(E.A)$ and (CLR) properties in the setting of complex valued metric spaces.

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The aim of this paper is to prove common fixed point theorems for integral type contractive condition using property $(E.A)$ and (CLR) property in complex valued metric spaces.

2. Preliminaries

Definition 2.1 ([9]). Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:
$z_1 \precsim z_2 \Leftrightarrow \mathrm{Re}(z_1) \leq \mathrm{Re}(z_2), \mathrm{Im}(z_1) \leq \mathrm{Im}(z_2)$.
Consequently, one can say that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

(1) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2), \mathrm{Im}(z_1) < \mathrm{Im}(z_2)$;
(2) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2), \mathrm{Im}(z_1) = \mathrm{Im}(z_2)$;
(3) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2), \mathrm{Im}(z_1) < \mathrm{Im}(z_2)$;
(4) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2), \mathrm{Im}(z_1) = \mathrm{Im}(z_2)$.

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (1)–(3) is satisfied and we will write $z_1 \prec z_2$ if only (3) is satisfied.

Note that one can easily verifies that

- $a, b \in R$ and $a \leq b \Rightarrow az \precsim bz$ for all $z \in \mathbb{C}$;
- $0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|$;
- $z_1 \precsim z_2$ and $z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

Definition 2.2 ([9]). Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$ satisfies the following axioms:

(1) $0 \precsim d(x, y)$, for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;
(2) $d(x, y) = d(y, x)$, for all $x, y \in X$;
(3) $d(x, y) \precsim d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a complex valued metric on X and the pair (X, d) is called complex valued metric space.

Example 2.1 ([13]). Let $X = \mathbb{C}$ and $d : X \times X \to \mathbb{C}$ be the mapping defined by

$$d(x, y) = e^{im}|x - y|,$$

where $x, y \in X$ and $0 \leq m \leq \frac{\pi}{2}$. Then (X, d) is a complex valued metric space.

Definition 2.3 ([9]). Let $\{x_n\}$ be a sequence in complex valued metric (X, d) and $x \in X$. Then x is called the limit of $\{x_n\}$ if for every $c \in \mathbb{C}$, with $0 < c$ there is $n_0 \in N$ such that $d(x_n, x) < c$ for all $n > n_0$ and we write $\lim_{n\to\infty} x_n = x$.

Lemma 2.1 ([9]). *Any sequence $\{x_n\}$ in complex valued metric space (X, d) converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

Definition 2.4 ([14]). Let K and L be self maps of a non empty set X. Then

(i) $x \in X$ is said to be fixed point of L if $Lx = x$.
(ii) $x \in X$ is said to be a coincidence point of K and L if $Kx = Lx$.
(iii) $x \in X$ is said to be a common fixed point of K and L if $Kx = Lx = x$.

Definition 2.5 ([10]). Let X be a complex valued metric space. Then the self-mappings $K, L : X \to X$ are weakly compatible if there exist a point $x \in X$ such that $KLx = LKx$ whenever $Kx = Lx$.

Definition 2.6 ([11]). Two self-maps K and L on a complex valued metric space X satisfy property $(E.A)$, if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Kx_n = \lim_{n\to\infty} Lx_n = x \quad \text{for some } x \in X.$$