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Original article

Spectral estimates of the p-Laplace Neumann operator in conformal regular domains

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Abstract

In this paper we study spectral estimates of the p-Laplace Neumann operator in conformal regular domains $\Omega \subset \mathbb{R}^2$. This study is based on (weighted) Poincaré–Sobolev inequalities. The main technical tool is the theory of composition operators in relation with the Brennan's conjecture. We prove that if the Brennan's conjecture holds for any $p \in (4/3, 2)$ and $r \in (1, p/(2-p))$ then the weighted (r, p)-Poincare–Sobolev inequality holds with the constant depending on the conformal geometry of Ω. As a consequence we obtain classical Poincare–Sobolev inequalities and spectral estimates for the first nontrivial eigenvalue of the p-Laplace Neumann operator for conformal regular domains.

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1. Introduction and methodology

Let $\Omega \subset \mathbb{R}^2$ be a simply connected planar domain with a smooth boundary $\partial\Omega$. We consider the Neumann eigenvalue problem for the p-Laplace operator $(1 < p < 2)$:

$$\begin{cases} -\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = \mu_p |u|^{p-2} u & \text{in } \Omega \\ \dfrac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.1}$$

The weak statement of this spectral problem is as follows: a function u solves the previous problem if and only if $u \in W^{1,p}(\Omega)$ and

$$\iint_\Omega \left(|\nabla u(x,y)|^{p-2}\nabla u(x,y)\right) \cdot \nabla v(x,y)\, dxdy = \mu_p \iint_\Omega |u|^{p-2} u(x,y) v(x,y)\, dxdy$$

for all $v \in W^{1,p}(\Omega)$.

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The first nontrivial Neumann eigenvalue μ_p can be characterized as

$$\mu_p(\Omega) = \min\left\{ \frac{\iint_\Omega |\nabla u(x,y)|^p\, dxdy}{\iint_\Omega |u(x,y)|^p\, dxdy} : u \in W^{1,p}(\Omega) \setminus \{0\},\ \iint_\Omega |u|^{p-2} u\, dxdy = 0 \right\}.$$

Moreover, $\mu_p(\Omega)^{-\frac{1}{p}}$ is the best constant $B_{p,p}(\Omega)$ (see, for example, [1,2]) in the following Poincaré–Sobolev inequality

$$\inf_{c\in\mathbb{R}} \|f - c \mid L^p(\Omega)\| \leq B_{p,p}(\Omega) \|\nabla f \mid L^p(\Omega)\|, \quad f \in W^{1,p}(\Omega).$$

We prove, that $\mu_p(\Omega)$ depends on the conformal geometry of Ω and can be estimated in terms of Sobolev norms of a conformal mapping of the unit disc $\mathbb{D}$ onto Ω (Theorem A).

The main technical tool is existence of universal weighted Poincaré–Sobolev inequalities

$$\inf_{c\in\mathbb{R}} \left(\iint_\Omega |f(x,y) - c|^r h(x,y)\, dxdy \right)^{\frac{1}{r}} \leq B_{r,p}(\Omega, h) \left(\iint_\Omega |\nabla f(x,y)|^p\, dxdy \right)^{\frac{1}{p}}, \quad f \in W^{1,p}(\Omega), \tag{1.2}$$

in any simply connected planar domain $\Omega \neq \mathbb{R}^2$ for conformal weights $h(x,y) := J_\varphi(x,y) = |\varphi'(x,y)|^2$ induced by conformal homeomorphisms $\varphi : \Omega \to \mathbb{D}$.

Main results of this article can be divided onto two parts. The first part is the technical one and concerns weighted Poincaré–Sobolev inequalities in arbitrary simply connected planar domains with nonempty boundaries (Theorem C *and its consequences). Results of the first part will be used for (non weighted) Poincaré–Sobolev inequalities in so-called conformal regular domains (Theorem* B*) that lead to lower estimates for the first nontrivial eigenvalue μ_p (Theorem* A*). To the best of our knowledge lower estimates were known before for convex domains only. The class of conformal regular domains is much larger. It includes, for example, bounded domains with Lipschitz boundaries and quasidiscs, i.e images of discs under quasiconformal homeomorphisms of whole plane.*

Brennan's conjecture [3] is that for a conformal mapping $\varphi : \Omega \to \mathbb{D}$

$$\iint_\Omega |\varphi'(x,y)|^\beta\, dxdy < +\infty, \quad \text{for all } \frac{4}{3} < \beta < 4. \tag{1.3}$$

For the inverse conformal mapping $\psi = \varphi^{-1} : \mathbb{D} \to \Omega$ Brennan's conjecture [3] states

$$\iint_{\mathbb{D}} |\psi'(u,v)|^\alpha\, dudv < +\infty, \quad \text{for all } -2 < \alpha < \frac{2}{3}. \tag{1.4}$$

A connection between Brennan's Conjecture and composition operators on Sobolev spaces was established in [4]:

Equivalence Theorem. *Brennan's Conjecture* (1.3) *holds for a number $\beta \in (4/3; 4)$ if and only if a conformal mapping $\varphi : \Omega \to \mathbb{D}$ induces a bounded composition operator*

$$\varphi^* : L^{1,p}(\mathbb{D}) \to L^{1,q(p,\beta)}(\Omega)$$

for any $p \in (2; +\infty)$ and $q(p,\beta) = p\beta/(p + \beta - 2)$.

The inverse Brennan's Conjecture states that for any conformal mapping $\psi : \mathbb{D} \to \Omega$, the derivative ψ' belongs to the Lebesgue space $L^\alpha(\mathbb{D})$, for $-2 < \alpha < 2/3$. The integrability of the derivative in the power greater than $2/3$ requires some restrictions on the geometry of Ω. If $\Omega \subset \mathbb{R}^2$ is a simply connected planar domain of finite area, then

$$\iint_{\mathbb{D}} |\psi'(u,v)|^2\, dudv = \iint_{\mathbb{D}} J_\psi(u,v)\, dudv = |\Omega| < \infty.$$

Integrability of the derivative in the power $\alpha > 2$ is impossible without additional assumptions on the geometry of Ω. For example, for any $\alpha > 2$ the domain Ω necessarily has a finite geodesic diameter [5].

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