

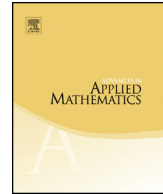


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Context-free grammars for permutations and increasing trees



William Y.C. Chen^a, Amy M. Fu^b

^a Center for Applied Mathematics, Tianjin University, Tianjin 300072, PR China

^b Center for Combinatorics, LPMC, Nankai University, Tianjin 300071, PR China

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ABSTRACT

We introduce the notion of a grammatical labeling to describe a recursive process of generating combinatorial objects based on a context-free grammar. By labeling the ascents and descents of Stirling permutations, we obtain a grammar for the second-order Eulerian polynomials. Using the grammar for 0-1-2 increasing trees given by Dumont, we obtain a grammatical derivation of the generating function of the André polynomials obtained by Foata and Schützenberger. We also find a grammar for the number $T(n, k)$ of permutations on $[n] = \{1, 2, \dots, n\}$ with k exterior peaks. We demonstrate that Gessel's formula for the generating function of $T(n, k)$ can be deduced from this grammar. From a grammatical point of view, it is easily seen that the number of the permutations on $[n]$ with k exterior peaks equals the number of increasing trees on $\{0, 1, 2, \dots, n\}$ with $2k + 1$ vertices of even degree. We present a combinatorial proof of this fact, which is in the spirit of the recursive construction of the correspondence between even increasing trees and up-down permutations, due to Kuznetsov, Pak and Postnikov.

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E-mail addresses: chenyc@tju.edu.cn (W.Y.C. Chen), fu@nankai.edu.cn (A.M. Fu).

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1. Introduction

In this paper, a *context-free grammar* G over a set $V = \{x, y, z, \dots\}$ of variables is a set of substitution rules replacing a variable in V by a Laurent polynomial of variables in V . For a context-free grammar G over V , the formal derivative D (introduced in [2]) with respect to G is defined as a linear operator acting on Laurent polynomials with variables in V such that each substitution rule is treated as the common differential rule that satisfies the following relations,

$$D(u + v) = D(u) + D(v),$$

$$D(uv) = D(u)v + uD(v).$$

For a constant c , we have $D(c) = 0$. Clearly, the Leibniz formula is also valid:

$$D^n(uv) = \sum_{k=0}^n \binom{n}{k} D^k(u)D^{n-k}(v).$$

Since $D(w w^{-1}) = 0$, we have

$$D(w^{-1}) = -\frac{D(w)}{w^2}.$$

A formal derivative D is also associated with an exponential generating function. For a Laurent polynomial w of variables in V , let

$$\text{Gen}(w, t) = \sum_{n \geq 0} D^n(w) \frac{t^n}{n!}.$$

Then we have the following relations:

$$\text{Gen}'(w, t) = \text{Gen}(D(w), t), \tag{1.1}$$

$$\text{Gen}(u + v, t) = \text{Gen}(u, t) + \text{Gen}(v, t), \tag{1.2}$$

$$\text{Gen}(uv, t) = \text{Gen}(u, t)\text{Gen}(v, t), \tag{1.3}$$

where u, v and w are Laurent polynomials of variables in V and $\text{Gen}'(w, t)$ stands for the derivative of $\text{Gen}(w, t)$ with respect to t .

To illustrate the connection between context-free grammars and combinatorial structures, we recall the following grammar introduced by Dumont [4]:

$$G: \quad x \rightarrow xy, \quad y \rightarrow xy. \tag{1.4}$$

He showed that it generates the *Eulerian polynomials* $A_n(x)$. Let S_n denote the set of permutations on $[n] = \{1, 2, \dots, n\}$. For a permutation $\pi = \pi_1 \pi_2 \cdots \pi_n \in S_n$, an index

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